24th GAMM Seminar on Microstructures 2025

All talks take place in **Festsaal Luisenstraße (Luisenstraße 56, 10117 Berlin)**

Thursday, January 30, 2025

Young Researchers' Meeting

08:30 - 09:00		**Registration**
09:00	**Opening**	
	Antonio Tribuzio	Rigidity and flexibility in martensitic materials: the Tartar square
	Christina Völlmecke	Make – Break – S(t)imulate: the joy of interdisciplinary Mechanics
10:20	**Coffee break**	
10:40	Matthias Grützner	On a variational model for microstructures in shape-memory alloys: energy scaling behaviour
	Sarah Dinkelacker-Steinhoff	A variational model of microstructure evolution with multi-phase transformations in elasto-plastic materials
	Emilie Pirch	Finite element schemes for the Monge-Ampère equation
	Alexander Niehüser	Efficient algorithms for crystal plasticity theory based on augmented Lagrangians
	Moritz Gau	Viscoelastic phase separation: Well-posedness and singular limit to viscous Cahn–Hilliard equation
	Mert Bastug	Neumann's sieve with random perforations
	Michele Aldé	BDF2-type integrator for Landau-Lifshitz-Gilbert equation in micromagnetics
12:30	**Closing**	of the young researchers' meeting

GAMM Seminar

12:30 - 13:30		**Registration**
13:30	**Opening**	
13:35	Yiqian He	A variational effective model for multiscale damage analysis
13:55	Oliver Suchan	Micro-macro coupling for optimizing scaffold mediated bone regeneration
14:15	Andera Chiesa	Finite-strain viscoelastic evolution and accretive phase-change
14:35	Anne Sur	A variational phase-field model for ductile fracture considering stress triaxiality effects
14:55	**Coffee break**	
15:30	Anna Pandolfi	Computational continuum and micromechanical models for the human cornea
16:15	Robert Lasarzik	Recent Results on Flows in Anisotropic Media
16:35	Eduard Rohan	Homogenization based two-scale modelling of porous inflatable structures with nonlinear interactions
16:55	David Wiedemann	Polarization filters as homogenization limits for Maxwell's equations
17:30		**Meeting of the GAMM Activity Group**
19:30		**Dinner at *Fischer und Lustig*,** Poststr. 26, 10178 Berlin

Friday, January 31, 2025

09:00	Benedikt Wirth	Microstructure emerging in shape optimization of elastic structures
09:45	Oliver Sander	Simulation and Numerical Analysis of Wrinkle Patterns on Coated Substrates
10:05	Camillo Tissot	On scaling properties for a class of two-well problems for higher order homogeneous linear differential operators
10:25	**Coffee break**	
10:55	Georg Heinze	Gradient flows on metric graphs with reservoirs: Microscopic derivation and multiscale limits
11:15	Lennart Machill	Positive temperature in nonlinear thermoviscoelasticity and the derivation of linearized models
11:35	Amartya Chakrabortty	Asymptotic analysis of periodic composite plate with external loadings and matrix pre-strain
11:55	Martin Heida	Upscaling Battery Models: Connecting Microscopic Dynamics with Macroscopic Behavior
12:15	**Break**	
13:15	Janusz Ginster	A scaling law for a model of epitaxially strained elastic films with dislocations
13:35	Timo Neumeier	Dimension reduction in computational polyconvexification for isotropic functions
13:55	Maximilian Köhler	Concurrent numerical rank-one convexification and Young measure evolution via Wasserstein-1 distance in continuum damage mechanics
14:15	**Coffee break**	
14:45	Michele Ruggeri	Finite element methods for magnetoelastic materials
15:05	Ruming Zhang	A novel method to analyse the radiation condition for the scattering problems with periodic media
15:25	Johannes Storn	Guaranteed upper bounds for the iteration error and modified iterative schemes via discrete duality
15:45	Ngoc Tien Tran	Hybrid high-order methods for convex minimization problems
16:05	**Closing**	

Abstracts

Michele Aldé (TU Wien): *BDF2-type integrator for Landau-Lifshitz-Gilbert equation in micromagnetics*

We consider the Landau-Lifshitz-Gilbert equation (LLG) that models time-dependent micromagnetic phenomena. We propose a full discretization that employs first-order finite elements in space and a BDF2-type two-step method in time. In each time step, only one linear system of equations has to be solved. We prove that the integrator guarantees unconditional weak convergence in H^1 to weak solutions of LLG. Moreover, if a smooth strong solution exists, then a-priori error estimates guarantee $\mathcal{O}(k^2 + h)$, where k is the time-step size and h is the spatial mesh-size, i.e., first-order in space and second-order in time. Numerical experiments conclude the talk.

Mert Bastug (Universität Münster): *Neumann's sieve with random perforations*

We study the homogenization of the Poisson equation in a bounded domain divided into two regions by a perforated hyperplane, a problem commonly known as Neumann's sieve. This is a simplified model for the flow of a fluid through a perforated wall with zero thickness. The perforations have random radii and are distributed randomly along the hyperplane. We establish homogenization for a class of stationary marked point processes generating the centers and the radii of the perforations. We obtain our results under minimal assumptions, requiring only that the average capacity of the perforations is finite. Notably, our framework does not impose restrictions on the size of the radii or the minimum distance between perforation centers, allowing the perforations to form clusters with high probability. Despite this, we demonstrate that these clusters do not influence the homogenized equation.

Amartya Chakrabortty (RPTU Kaiserslautern and Landau & Fraunhofer Institute of Industrial Mathematics): *Asymptotic analysis of periodic composite plate with external loadings and matrix pre-strain*

A study on the simultaneous homogenization and dimension reduction of high-contrast periodic composite plates within the framework of non-linear elasticity is presented. The periodic thin composite plate is characterized by two parameters: thickness $\delta \in (0,1)$ and period $\varepsilon \in (0,1)$. Two asymptotic analysis are considered: dimension reduction as $\delta \to 0$ and homogenization as $\varepsilon \to 0$. When both parameters tend zero simultaneously, this process is referred to as simultaneous homogenization and dimension reduction $(\varepsilon, \delta) \to (0,0)$. Assuming the limit

$$\lim_{(\varepsilon,\delta)\to(0,0)} \frac{\delta}{\varepsilon} = \kappa \in [0, 1/2)$$

exists for the whole sequence, two cases are considered. First, for $\kappa \in (0, 1/2)$, a simply connected periodic perforated plate with holes filled by a relatively soft material is considered. The relationship between the stiff and soft elastic coefficients is ε^2. The applied forces are scaled to get the total energy in the Kármán-type regime. An extension operator for the stiff part is presented. The use of a re-scaling unfolding operator allows for deriving the asymptotic behavior of the Green St. Venant's strain tensor in terms of displacements. The limit homogenized energy with linear elastic cell problems is established using the Γ-convergence. Finally, the loading is replaced by a pre-strain, with results documented in [1]. Second, for $\kappa = 0$, the soft part of the plate is considered to be simply connected and the stiff subdomains are disjoint, as discussed in [2]. The total energy of the system is again in the K´arm´an regime type, which is obtained by scaling the applied forces and using the geometric rigidity for a star-shaped domain. The limit behavior of the Green-St. Venant's strain tensor is determined using three types of unfolding operators. Using Γ-convergence, the limit homogenized energy is derived, and the existence of a minimizer is proven.

These works are conducted as part of my PhD research under the esteemed supervision of Prof. Dr. Georges Griso (Sorbonne University, Paris) and Dr. Julia Orlik (Fraunhofer ITWM, Kaiserslautern).

References:

[1] A. Chakrabortty, G. Griso, and J. Orlik. "Dimension reduction and homogenization of composite plate with matrix pre-strain." Asymptotic Analysis Preprint (2024): 1-56.

[2] A. Chakrabortty, G. Griso, and J. Orlik. "Dimension reduction and homogenization for soft plates with printed disconnected rigid substructures" (In preparation)

Andrea Chiesa (Universität Wien): *Finite-strain viscoelastic evolution and accretive phase-change*

We investigate the evolution of a two-phase viscoelastic material at finite strains, where one phase accretes in time in its normal direction at the expense of the other. Mechanical response depends on the phase, and growth is influenced by the mechanical state at the boundary of the accreting phase, making the model fully coupled. This setting is inspired by the early stage development of solid tumours, as well as by the swelling of polymer gels. Both a diffused- and a sharp-interface variant of the model are proved to admit solutions and the sharp-interface limit is investigated.

Sarah Dinkelacker-Steinhoff (Ruhr-Universität Bochum): *A variational model of microstructure evolution with multi-phase transformations in elasto-plastic materials*

There are several models describing microstructural evolution of elasto-plastic materials in terms of lamination. In our work, introduced in [1], we combine an underlying probability matrix that governs the transition between different states with the so-called dissipation distance to obtain multi-phase transformations in the general sense. Together with fixed volume fractions implemented by Young measures, the model is able to predict an instantaneous transition in an incremental updating process.

References:

[1] Dinkelacker-Steinhoff, S., Hackl, K. (2024). A general model of microstructure evolution including phase transformation in elasto-plastic materials. Proceedings in Applied Mathematics and Mechanics, 24, e202400024. https://doi.org/10.1002/pamm.202400024

Moritz Gau (WIAS Berlin): *Viscoelastic Phase Separation: Well-posedness and Singular Limit to Viscous Cahn–Hilliard Equation*

Viscoelastic phase separation plays an important role in biological cells, for instance, RNA or proteins can undergo phase separation to form membraneless condensates, which is crucial for biological functions. In our contribution, we consider a model for phase separation in poly- mer solutions consistent with the second law of thermodynamics, introduced by Zhou et al. 2006. For the full model with additional stress diffusion, existence of global-in-time weak solu- tions is established by Brunk–Lukáčová-Medvid'ová 2022 via a Faedo–Galerkin ansatz for both degenerate and non-degenerate mobilities.
In this talk, we neglect the hydrodynamic transport and consider a constant mobility and a regular potential. Moreover, we focus on the case of a constant bulk modulus and a constant relaxation time. Exploiting the gradient-flow structure, we establish the global well-posedness (meaning existence, uniqueness and stability estimate) of the initial-boundary-value problem using a time-incremental minimisation scheme. We then address a singular limit through evolutionary Γ-convergence and Evolutionary Variational Inequality (EVI) solutions, tackling the primary challenge posed by the lack of equi-compactness of the energies.
This is a joint work with Katharina Hopf (WIAS) and Matthias Liero (WIAS).

Janusz Ginster (WIAS Berlin): *A scaling law for a model of epitaxially strained elastic films with dislocations*

Epitaxy is a special form of crystal growth and of great importance in modern technology. We consider a crystalline film that is deposited on a (rigid) substrate. The misfit between the crystal structure of the film and the substrate can lead to dislocations and can have an influence on the morphology of the film. In this talk, we will study a static variational model which is based on linearized elasticity and takes into account the surface energy of the film's free surface as well as the nucleation energy of the dislocations. We will discuss a scaling law for the infimum of the energy. This result quantifies that in certain parameter regimes the formation of islands or dislocations is expected. This is joint work with L. Abel and B. Zwicknagl.

Matthias Grützner (ETH Zürich): On a variational model for microstructures in shape-memory alloys: energy scaling behaviour

We study a geometrically linearised variational model motivated through pattern formation during certain solid-solid phase transformations in shape-memory alloys. First, we show how the geometrically linearised model may be obtained through a second order Taylor expansion of the non-linear energy introducing a tilted notion of symmetry of matrices. We see that the approximative quality of the geometrically linearised energy is only strong for nearly symmetric matrices and fails if the skew-symmetric part dominates. The main part then concerns an energy scaling law for the geometrically linearised energy. Through construction of a specific function for the upper bound, containing branching microstructures, as well as an ansatz-free lower bound of the energy, we obtain a scaling law for sufficiently small $\epsilon \in \mathbb{R}^+$ of the form

$$C_1\epsilon^{4/5} \leq E^\epsilon \leq C_2\epsilon^{4/5}. \tag{1}$$

This agrees with results known for the non-linear energy. Further, we show that a solution exists such that the minimal energy is indeed attained.

Yiquian He (Dalian University of Technology, China): *A variational effective model for multiscale damage analysis*
Authors: X. Xu, G. Jezdanb, H. Yanga,Y. He, and K. Hackl

In the damage analysis of material with complex microstructures, a direct numerical simulation (DNS) could produce huge computational costs. Alternatively, the multiscale modeling is a more effective method by using less degrees of freedom to balance the computational accuracy and cost. In this paper, a new multiscale method for damage analysis is proposed based on the framework of the variational effective model for elastoplastic problems originally presented in author's previous work. The key idea is to construct variational statement for the free energy and the dissipation potential of a coarse scale model by relating the free energy and the dissipation potential of a fine scale model. In this way, the damage analysis can be directly simulated at the coarse scale solution, resulting in significantly higher computational efficiency compared to the DNS in fine scale. In addition, the relaxation damage model is used to effectively avoid ill-posed boundary value problems in damage analysis without use of gradients or integration techniques. Three numerical examples are provided to demonstrate the effectiveness of the proposed method, by comparing the computational accuracy and cost of the proposed method with reference solutions provided by DNS and FE^2 algorithm.

Martin Heida (WIAS Berlin): *Upscaling Battery Models: Connecting Microscopic Dynamics with Macroscopic Behavior*

The relationship between voltage, stored charge, and applied current is the primary observable that provides insights into the inner workings of a battery. Our goal is to understand these observed relationships by upscaling a simple PDE from the microscopic model. This process leads to a forward-backward parabolic equation that captures the macroscopic behavior of the battery. By employing a sophisticated argument based on established analytical results, we bridge the gap between microscopic modeling and experimental macroscopic observations. This is joint work with Manuel Landstorfer.

Georg Heinze (WIAS Berlin): *Gradient flows on metric graphs with reservoirs: Microscopic derivation and multiscale limits*

I will discuss evolution equations on metric graphs with reservoirs, that is graphs where a one-dimensional interval is associated to each edge and, in addition, the vertices are able to store and exchange mass with these intervals. Focusing on the case where the dynamics are driven by an entropy functional defined both on the metric edges and vertices, we provide a rigorous understanding of such coupled systems of ordinary and partial differential equations as (generalized) gradient flows in continuity equation format. Approximating the edges by a sequence of vertices, which yields a fully discrete system, we establish existence of solutions in this formalism. Furthermore, we study several scaling limits using the recently developed framework of EDP convergence with embeddings to prove convergence to gradient flows on reduced metric and combinatorial graphs. Finally, numerical studies confirm our theoretical findings and provide additional insights into the dynamics under rescaling. This is joint work with Jan-Frederik Pietschmann and André Schlichting.

Maximilian Köhler (Ruhr-Universität Bochum): *Concurrent Numerical Rank-One Convexification and Young Measure Evolution via Wasserstein-1 Distance in Continuum Damage Mechanics*
Authors: M. Köhler, T. Neumeier, M. A. Peter, D. Peterseim, D. Balzani

The relaxation of problems involving non-convex potentials through (semi-)convexification is an appealing regularization technique for two key reasons. First, it can restore the existence of minimizers without introducing additional length scales or auxiliary PDEs. Second, it enables the description of microstructures at the material point in the form of Young measures. However, this approach presents two significant challenges. On one hand, the numerical (semi-)convexification process is computationally demanding and requires the development of carefully tailored algorithms. On the other hand, modeling the evolution of microstructures—particularly laminates of arbitrary depth—is inherently complex. This talk addresses these challenges in two parts. First, it introduces an algorithm designed for the upper-bound approximation of the rank-one convex envelope, integrated within three-dimensional finite element frameworks, thereby addressing the challenge of relaxing the original 1-D continuum damage mechanics model. Second, it presents a novel approach for numerically modeling the evolution of microstructures using the Wasserstein-1 distance.

Robert Lasarzik (WIAS Berlin): *Recent Results on Flows in Anisotropic Media*

In this talk, we present recent advances in the study of anisotropic fluid flows. Specifically, we investigate a fluid containing dissolved charged particles with a prescribed anisotropic structure. Unlike isotropic fluids, anisotropic fluids exhibit large-scale flow when subjected to highly oscillating electric fields, a phenomenon that could be exploited for fluid control in small-scale applications, such as microfluidic devices. We provide a comprehensive mathematical analysis of the underlying system and compare our numerical simulations with experimental data. Furthermore, we introduce a novel norm-preserving numerical scheme for nematic liquid crystals and establish the convergence of approximate solutions toward energy-variational solutions. Computational studies are conducted to demonstrate the practical applicability of the proposed algorithm.

Lennart Machill (Universität Bonn): *Positive temperature in nonlinear thermoviscoelasticity and the derivation of linearized models*

According to the Nernst theorem or, equivalently, the third law of thermodynamics, the absolute zero temperature is not attainable. Starting with an initial positive temperature, we show that there exist solutions to a Kelvin-Voigt model for quasi-static nonlinear thermoviscoelasticity at a finite-strain setting [Mielke-Roubíček '20], obeying an exponential-in-time lower bound on the temperature. Afterwards, we focus on the case of deformations near the identity and temperatures near a critical positive temperature, and we show that weak solutions of the nonlinear system converge in a suitable sense to solutions of a system in linearized thermoviscoelasticity. Our result extends the recent linearization result in [Badal-Friedrich-Kružík '23], as it allows the critical temperature to be positive. The talk is based on the preprint `https://arxiv.org/abs/2407.02035`.

Timo Neumeier (Universität Augsburg): *Dimension reduction in computational polyconvexification for isotropic functions*

The simulation of non-linear variational models in computational mechanics with non-convex energy densities, such as those arising in damage models where damage evolution drives the formation of non-convexity, often suffers from mesh dependence and computational instability. Relaxation techniques by means of semi-convexification of the energy density provide mesh-independent approximations, improved stability, and insights into the microstructural evolution of the material, making them valuable for the simulation of boundary value problems, e.g., in continuum damage mechanics. A significant challenge, however, is the computational effort required to compute semi-convex envelopes. In this talk, we address the efficient approximation of the polyconvex envelope. Using the signed singular value characterisation of isotropic functions, the polyconvexification process transitions from the lifted $d \times d$- deformation gradient space to a d-dimensional manifold in the minors-lifted space of signed singular values. This dimension reduction, combined with well-known optimisation and computational geometry algorithms, significantly enhances computational efficiency. Numerical experiments demonstrate substantial speed-ups in computational time and indicate the feasibility of this relaxation approach for robust finite element simulations. joint work with: L. Balazi, D. Balzani, M. Köhler, M. A. Peter, D. Peterseim, D. Wiedemann

Alexander Niehüser (TU Dortmund): *Efficient algorithms for crystal plasticity theory based on augmented Lagrangians*

Polycrystalline materials like dual-phase steels are very promising for different industrial applications. Their favorable macroscopic properties are governed by deformation and damage mechanisms at the microscale. In order to perform simulations of such polycrystalline materials, a robust and efficient crystal plasticity formulation is essential. The augmented Lagrangian algorithm for crystal plasticity leads to a very robust formulation. However, it is numerically very extensive. In this contribution, a novel improved augmented Lagrangian algorithm for crystal plasticity is presented and its properties are analyzed. Several numerical examples show that this novel algorithm is indeed faster and – at the same time – also more robust than the original Lagrangian formulation.

Anna Pandolfi (Politecnico Milano): *C*omputational Continuum and Micromechanical Models for the Human Cornea

I will provide an overview of the research developed in our group in terms of computational models for the human cornea. The eye is a multi-component biological system, where mechanics, optics, transport phenomena and chemistry are strictly interlaced. The eye's response to external action is patient-specific and it can be predicted only by a customized approach, that accounts for the multiple physics and for the intrinsic microstructure of the tissues, developed with the aid of computational biomechanics. Our activity in the last years has been devoted to the development of a comprehensive patient-specific model of the cornea, able to simulate refractive intervention such as LASIK and SMILE. While the geometrical aspects are fully under control, the major difficulties are related to the characterization of the tissues, which require the definition of clinical in-vivo tests to complement known results of multiple ex-vivo tests. The interpretation of in-vivo tests is very complex, since the entire structure of the eye is involved and the characterization of the single tissue is not trivial, therefore the availability of micromechanical models constructed from the information given by diagnostic images of the eye represents a fundamental support for the characterization of the corneal tissues, especially in the case of pathologic conditions.

Emilie Pirch (Friedrich-Schiller-Universität Jena): *Finite element schemes for the Monge-Ampère equation*

The Monge-Ampère equation is a partial differential equation (PDE) that arises usually as part of a coupled system of equations and is frequently encountered in the context of transport or curvature prob-

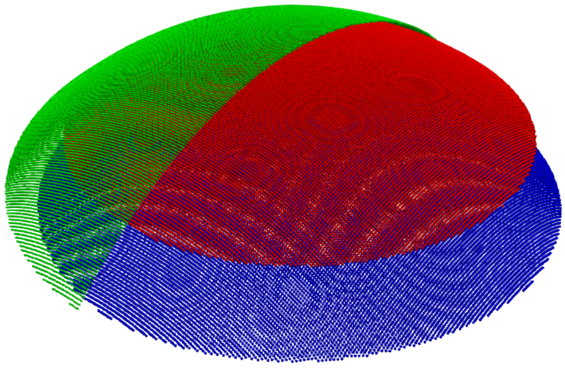

Figure 1: Patient specific geometry of a cornea, inclusive of anterior (green), posterior (blue) surfaces and refractive interface (red) for LASIK surgery.

lems. It is fully non-linear, degenerate elliptic and in non-divergence form and a special constraint is that the solution needs to be convex. All of these properties make it challenging to solve the Monge-Ampère equation numerically, as they are not intuitive with standard solution techniques, and recent advances in the numerical analysis of this PDE will be the subject of this talk. In particular, some finite element schemes using a regularization will be presented and one of the aims is to find an error estimator that allows adaptive computation. Furthermore, difficulties that arise from the limited availability of analytical tools will be discussed.

Eduard Rohan (University of West Bohemia, Pilsen): *Homogenization based two-scale modelling of porous inflatable structures with nonlinear interactions*
Authors: E. Rohan, V. Lukeš, J. Heczko

Recent advances in 3D printing technologies open new possibilities in design of porous metamaterials intended for fluid transport, or shape morphing with various potential applications. The talk is devoted to two kinds of nonlinearities featuring the fluid-solid interactions at the pore level of porous periodic structures. 1) The first one is related to the treatment of the problem imposed in the deformed configuration, such that the solid skeleton is hypoelastic. The homogenized model is derived under the small deformation assumption, however in the Eulerian frame. The nonlinearity associated with deforming configuration is respected by deformation-dependent homogenized coefficients of the poroelasticity. To reduce the computational efficiency, the sensitivity analysis of the homogenized coefficients with respect to deformation induced by the macroscopic quantities is employed. 2) The second related nonlinearity is caused by the unilateral "self-contact" interaction at the pore level, which yields the local variational inequalities. Since the fluid domain is affected by the contact closing the pore, the Stokes flow model requires a regularization to retain the pore connectivity. The macroscopic model attains the form of a nonlinear Biot continuum, whereby the Darcy flow model governs the fluid redistribution. In both the nonlinear problems, the limit two-scale models are derived using the periodic unfolding homogenization method applied to the consistent linearization of the incremental formulations. Both the problems are involved in the modelling of inflatable structures enabling also for fluid transport due to the peristaltic deformation. As an example we consider a metamaterial involving controlable piezoelectric segments. Numerical examples will be presented showing performance of the two-scale algorithms and features of the considered two-phase microstructures.

References:

E. Rohan, V. Lukeš, Homogenized model of peristaltic deformation driven flows in piezoelectric porous media, Computers & Structures, Volume 302, 2024, 107470, ISSN 0045-7949

E. Rohan, J. Heczko, Homogenization and numerical algorithms for two-scale modeling of porous media with self-contact in micropores. J. Comput. Appl. Math. 432: 115276 (2023)

Michele Ruggeri (University of Bologna): *F*inite element methods for magnetoelastic materials

Magnetoelastic materials are smart materials characterized by a strong interplay between their mechanical and magnetic properties. Due to their capability to change shape in response to applied magnetic fields, they currently find use in many technological applications requiring a magnetomechanical transducer, e.g., actuators and sensors. In this talk, we consider the finite element approximation of a nonlinear system of PDEs modeling the dynamics of magnetization and displacement in magnetoelastic materials in the small strain regime. We present a fully discrete structure-preserving numerical scheme and discuss its analysis (well-posedness, discrete energy law, stability, unconditional convergence). This is joint work with Hywel Normington (University of Strathclyde).

Oliver Sander (TU Dresden): *Simulation and Numerical Analysis of Wrinkle Patterns on Coated Substrates*

We consider a mechanical system consisting of an elastic substrate that has been coated by a stiffer layer while in a state of strain. The mismatch between the stress-free states of the substrate and the coating leads to the formation of wrinkle patterns, which are of interest both from a theoretical and an application point of view. We construct a model that combines a large-strain hyperelastic material with a geometrically exact Cosserat shell, and prove existence of solutions under reasonable assumption. We discretize the model using geometric finite elements, and prove existence of finite element solutions. Numerical examples show a good match between simulated to measured wrinkle patterns.

Oliver Suchan (Universität Freiburg): *M*icro-Macro Coupling for Optimizing Scaffold Mediated Bone Regeneration

When bone defects of large size occur, also called critical defects, the regenerating bone tissue is not able to bridge the resulting gap on its own and an additional artificial structure is necessary to support bone growth. We present a general framework to phenomenologically model three-dimensional transient scaffold- mediated bone regeneration and its associated optimization problem. By incorporating the microstructure into the model through periodic homogenization and the classical FE^2-method and an FE-FFT approach, we capture the effects of microscale fluctuations on the bone growth process. Numerical results and optimized scaffold designs that explicitly account for these microstructural variations are presented, demonstrating the potential of this approach for improving scaffold performance.

Johannes Storn (Universität Bielefeld): *G*uaranteed upper bounds for the iteration error and modified iterative schemes via discrete duality

We apply duality theory to discretized convex minimization problems to obtain computable guaranteed upper bounds for the distance of a given discrete function and the exact discrete minimizer. Furthermore, we show that the discrete duality framework extends convergence results of iterative schemes to a broader class of problems.

Anne Sur (TU Dresden): *A variational phase-field model for ductile fracture considering stress triaxiality effects*

Authors: A.-S. Sur, L. De Lorenzis, C. Maurini, O. S. Hopperstad

Ductile fracture is associated with plastic deformation and the development of voids in the material, which leads to a gradual fracture evolution. This work aims to capture the complex micromechanical processes such as void nucleation, growth and coalescence in a phenomenological sense. We provide a variational phase-field fracture model that includes triaxiality effects, i.e. captures the hydrostatic stresses arising under necking during plastic deformation. The hydrostatic stress is related to void nucleation and growth in the material, and is relevant for the prediction of ductile fracture initiation. The plasticity formulation is based on the modified Cam-Clay plasticity model that goes back to Roscoe and Burland [1]. It takes hydrostatic and deviatoric stresses into account and can therefore also handle more complex stress states in addition to standard von Mises plasticity. The behaviour of the proposed model is studied analytically and by various numerical simulations for high strength steel. These show for different benchmark tests that the model is suitable for predicting ductile fracture initiation and propagation.

References:

[1] K.H. Roscoe, J.B. Burland, On the Generalised Stress-strain Behaviour of 'Wet' Clay, In Engineering Plasticity (1968), J. Heyman and F.A. Leckie, Eds., Cambridge University Press, pp.535–609.

Camillo Tissot (Universität Bonn): *On Scaling Properties For A Class Of Two-Well Problems For Higher Order Homogeneous Linear Differential Operators*

We study the (lower) scaling behavior of a class of compatible two-well problems for higher order, homogeneous linear differential operators. To this end, we first deduce general lower scaling bounds which are determined by the vanishing order of the symbol of the operator on the unit sphere in the direction of the associated element in the wave cone. We complement the lower bound estimates by a analysis of the two-well problem for generalized (tensor-valued) symmetrized derivatives with the help of the (tensor-valued) Saint-Venant compatibility conditions. In two spatial dimensions for highly symmetric boundary data (but arbitrary tensor order $m \in \mathbb{N}$) we provide upper bound constructions matching the lower bound estimates. This illustrates that for the two-well problem for higher order operators new scaling laws emerge which are determined by the Fourier symbol in the direction of the wave cone.
Joint work with B. Raita, A. Rüland, and A. Tribuzio.

Antonio Tribuzio (Universität Bonn): *Rigidity and flexibility in martensitic materials: the Tartar square*

In recent years, the study of martensitic materials (such as certain alloys which display stunning mechanical properties like the shape-memory effect) led to the analysis of highly non-convex differential inclusions. Due to the lack of convexity, according to the prescribed regularity, there may be either many (flexibility) or one (rigidity) class of solutions. After introducing and motivating the problem, we try to find some information about the threshold regularity between rigidity and flexibility by studying a simplified (though extrimely intresting) toy-model, the so-called Tartar square, by relaxing the problem studying scaling laws of the related singularly perturbed elastic energy. The results presented in this talk are in collaboration with Angkana Rüland.

Ngoc Tien Tran (Universität Augsburg): The relaxation in the calculus of variation motivates the numerical analysis of a class of degenerate convex minimization problems with non-strictly convex energy densities with some convexity control and two-sided p-growth. The minimizers may be non-unique in the primal variable but lead to a unique stress. Examples include the p-Laplacian, an optimal design problem in topology optimization, and the convexified double-well problem. The approximation by hybrid high-order methods (HHO) utilizes a reconstruction of the gradients with piecewise Raviart-Thomas without stabilization on a regular triangulation into simplices. The application of this HHO method to the class of degenerate convex minimization problems allows for a unique conforming stress approximation. The main results are a priori and a posteriori error estimates for the stress error and a computable lower energy bound. Numerical benchmarks display higher convergence rates for higher polynomial degrees and include adaptive mesh-refining.

Christina Völlmecke (TU Berlin): *Make – Break – S(t)imulate: the joy of interdisciplinary Mechanics*

The talk outlines the methodologies and outcomes of interdisciplinary research conducted at the Make and Break Lab at Technische Universität Berlin. The lab integrates a three-tone of methods: Make (fabrication through 3D printing), Break (mechanical testing via tensile and compression experiments), and Simulate (semi-analytical and numerical analysis using open-source simulation tools). These methods are applied iteratively to research interdisciplinary problems at the intersection of materials science, engineering mechanics, and computational modeling.

The talk will highlight the applications and results achieved through collaborative efforts between researchers from diverse fields. Additionally, insights into further group activities stimulating innovation and outreach as well as an overview of the presenter's journey through academia will be given.

David Wiedemann (TU Dortmund): *Polarization filters as homogenization limits for Maxwell's equations*

Authors: B. Schweizer and D. Wiedemann

We consider the time harmonic Maxwell equations in a complex geometry that model polarization filters or Faraday cages. We study the situation that the domain of interest contains inclusions with infinite conductivity, the inclusions are distributed in a periodic fashion along a surface. The periodicity is $\eta > 0$ and the shape of the inclusion depends also on η since we want to model, e.g., thin structures. We are interested in the limit $\eta \to 0$ and in effective equations. Depending on geometric properties of the inclusions, the effective system can imply polarization or cancellation of the field.

Benedikt Wirth (Universität Münster): *Microstructure emerging in shape optimization of elastic structures*

So-called compliance minimization seeks the optimal geometry of an elastic material to withstand a fixed given mechanical load and, at the same time, to consume as little material as possible. In general this problem is ill-posed: Minimizers may not exist, and microstructure may form along minimizing sequences. However, for some externally applied loads and some loading geometries minimizers actually do exist, sometimes even without microstructure. Indeed, the problem features an interesting dichotomy: If the load has eigenstresses of opposite sign, minimizers do not exist, and the only optimal microstructure consists of a laminate. If both eigenstresses are of the same sign, there is suddenly a plethora of optimal microstructures, and even minimizers of fractal geometry are known. But is microstructure really necessary, and if yes, how does the microstructure of minimizers look like? We will discuss some answers to these problems. The work is joint with Peter Bella and Jonathan Fabiszisky.

Ruming Zhang (TU Berlin): *A novel method to analyse the radiation condition for the scattering problems with periodic media*

The radiation condition is the key topic in the analysis of scattering problems. Mathematically, a proper radiation condition guaratees the well-posedness of the problem; physically, a suitable radiation condition describes the physical process exactly. However, the topic is very challenging when the domain is periodic. It has already been proved that the scattered field satisfies $\frac{\partial u}{\partial r} - \imath k u = o\left(r^{-1/2}\right)$. The result is extended to periodic inhomogeneous layers.

In this talk, we will introduce a novel Floquet-Bloch transform based method to analyse the radiation condition. Based on the F-B transform, we decompose the scattered field into a Herglotz wave function and a fast decaying function. By analysing the radiation condition for the Herglotz wave function, we can show that $\frac{\partial u}{\partial r} - \imath k u = O\left(r^{-3/2}\right)$.