Summer Term 2025 Reading Seminar: Introduction to *p*-adic Hodge Theory

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There is a huge amount of mathematices going under the name '*p*-adic Hodge Theory'. One of the accepted core meanings of this expression is: the classification of local *p*-adic Galois representations. We take it here in this way (i.e. we do *not* look at the more geometric aspects of '*p*-adic Hodge Theory').

Thus, for a local field K of residue characteristic p, we are interested in the classification of continuous representations of $\operatorname{Gal}(\overline{K}/K)$ on finite dimensional \mathbb{Q}_p - or \mathbb{F}_p -vector spaces (with their *p*-adic resp. discrete topology). This is a huge field, with numerous competing concepts and theories. The aim of this seminar is to gain a robust understanding of the (most 'traditional') concepts of *Hodge-Tate*, of *de Rham*, of *semistable* and of *crystalline* $\operatorname{Gal}(\overline{K}/K)$ -representations, and of their *Hodge-Tate weights*.

We follow the text **Denis Benois: An introduction to** *p***-adic Hodge Theory.** All the following references are taken from these lecture notes. Talks 1-6 are preparatory. The core content to be memorized is in talks 7-11. We might consider assigning 120 minutes to each of these later talks.

Program

<u>1.</u> Give a selection from section 1. Omit the basics on local fields, but do include the following: Theorem 1.1.8, Theorem 1.3.11, Lemma 1.4.2, Proposition 1.6.3, Proposition 1.6.5.

<u>2.</u> Section 3. Important is Theorem 3.3.2.

3. Section 4. Important are Theorem 4.2.11 and Theorem 4.3.2 and Theorem 4.3.8.

<u>4.</u> Section 5. The material should be known to those of us who attended the seminar on (φ, Γ) -modules (and hence doesn't need to be treated here in full detail). Important are Proposition 5.3.3, Proposition 5.4.3, Theorem 5.4.4.

5. Subsections 6.1 and 7.1. Explain the meaning of Proposition 7.1.7.

- <u>6.</u> Section 8 up to Theorem 8.2.3.
- $\underline{7.}$ Section 9.
- $\underline{8.}$ Section 10.
- 9. Section 11. (You might omit subsubsection 11.3.1.)
- $\underline{10.}$ Section 12.

11. Subsections 13.1 and subsections 13.2, and more quickly subsections 13.3.