Summer Term 2024
Reading Seminar: p-adic Vertex Algebras

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Vertex Algebras are algebraic structures playing an important role in various approaches to the geometric Langlands Program, in finite group theory, in the study of modular forms, and, last but not least, in mathematical physics and conformal field theory. Until very recently, their study has been exclusively algebraic resp. archimedean-analytic. But the two recent papers [FM] and [K] propose to also investigate p-adic versions of vertex algebras. Our aim is to familiarize ourselves with this new idea. We will mostly focus on [FM], as this paper is apparently more palpable than [K], and as it also points out number theoretic applications, namely, links with p-adic modular forms.

The book [FB] starts with an extremely lucid introduction of (algebraic) Vertex Algebras, and this is how we will begin as well.

[FM] Cameron Franc, Geoffrey Mason: p-adic vertex operator algebras, Research in Number Theory 9(2) (2023)

[K] Victor G. Kac: Non-Archimedean Vertex Algebras (2023)

[FB] Edward Frenkel, David Ben-Zvi: Vertex Algebras and Algebraic Curves, AMS Mathematical Surveys and Monographs **88** (2004)

[B] Richard E. Borcherds: What is moonshine?

Program:

1. Laurent series, Fields, Locality

[FB] Chapter 1

2. The Heisenberg Vertex Algebra

[FB] Section 2.1 and 2.2; also give a brief sketch of section 2.3, but we should not be too detailed about proofs here.

3. p-adic Banach spaces

[FM] Section 3, Section 4 until Lemma 4.9

4. p-adic vertex algebras

[FM] (Section 2,) Section 4 from Definition 4.10, Section 5 until Definition 5.4

5. p-adic Goddard axioms

[FM] Section 5 from Theorem 5.5 until Lemma 5.11

6. p-adic conformal vertex algebras

[FM] Section 5 from Lemma 5.12, Section 6 until Lemma 6.6

7. Completion of algebraic vertex algebras

[FM] Section 6 from Definition 6.7, Section 7

8. p-adic locality

[FM] Section 8

9. The p-adic Heisenberg Algebra

[FM] Section 9; also, provide some background on p-adic modular forms, Eisenstein series, Dedekind's η-function as needed

10. Kummer congruences

[FM] Section 10; also, provide more background on Eisenstein series, Bernoulli numbers, Kummer congruences as needed

11. p-adic Moonshine

[FM] Section 11, consult also [B]