

READING SEMINAR, WINTER TERM 2025/2026  
**Integral  $p$ -adic Hodge theory: Prisms, the prismatic site**

*Integral  $p$ -adic Hodge theory* is the discipline which systematically investigates  $\mathbb{Z}_p$ -structures in objects coming up in *rational  $p$ -adic Hodge theory* (the latter being typically  $\mathbb{Q}_p$ -vector spaces). An exciting conceptual breakthrough has been achieved by Bharghav Bhatt and Peter Scholze in their discovery of *prisms* and the *prismatic site*. We try to understand the most basic elements of this theory.

We use the following two lecture note texts:

[B] B. Bhatt: Prismatic cohomology

<http://www-personal.umich.edu/~bhattb/teaching/prismatic-columbia/>

[K] K. Kedlaya: Notes on prismatic cohomology

1.  $\delta$ -rings

[B] sections 1 and 2 and Lemma 3.3 of lecture II.

2. Rings of Witt vectors; perfect prisms

[K] par. 3 (cf. also [B] section 2 of lecture II). One could also consult other texts on Witt vectors. (Note: By 'Witt vectors' we mean here what sometimes are referred to as ' $p$ -typical' Witt vectors (as opposed to 'big' Witt vectors).)

3. Distinguished elements

[B] section 1 of lecture III

4. Big Witt vectors, Prisms

[K] par. 4 and [B] section 3 of lecture III

5. Derived completeness

[K] par 6, cf. also [B] section 2 of lecture III.

6. Perfect prisms

[K] par. 7

7. Lenses

[K] section 8.1 — 8.3

8. Gluing Lenses; Homotopy categories

This talk has two disjoint parts: First, [K] section 8.3. Then, a reminder on homotopy categories, conveniently following [K] sections 9.1 — 9.4.

9. Derived categories

[K] section 9.5 and par. 10

10. The prismatic site

[K] par. 11

11. The Hodge-Tate comparison map

[K] par. 12

12. More

[K] par. 13 — 29: Choose!