Introduction to the AdS/CFT correspondence
HW set 1

April 13, 2012

1 Reading

Read pages 1-30 from Martin, pages 1-14 from Lykken and pages 17-34 from Quevedo.

2 Important spinor identities

Prove that

$$\xi\chi \equiv \xi^\alpha\chi_\alpha = \chi^\beta\xi_\beta \equiv \chi\xi \tag{1}$$

with no minus sign!

$$\xi^\dagger\bar{\sigma}^\mu\chi = -\chi\sigma^\mu\xi^\dagger \tag{2}$$

$$\xi\sigma^\mu\bar{\sigma}^\nu\chi \;=\; \chi\sigma^\nu\bar{\sigma}^\mu\xi \tag{3}$$

and the Fierz rearrangement identity:

$$\chi_\alpha\,(\xi\eta) = -\xi_\alpha\,(\eta\chi) - \eta_\alpha\,(\chi\xi), \tag{4}$$

3 The number of bosons equals the number of fermions

Prove that in any supersymmetric multiplet, the number n_B of bosons equals the number n_F of fermions,

$$n_B \;=\; n_F$$

following the steps bellow:

- Define the *fermion "number" operator* $(-1)^F = (-)^F$ such that

$$(-)^F\,|B\rangle \;=\; |B\rangle\;, \qquad (-)^F\,|F\rangle \;=\; -|F\rangle\;.$$

- Show that $(-)^F$ anticommutes with Q_α

$$\Big\{(-)^F\;,\;Q_\alpha\Big\} \;=\; 0\;.$$

- Using the cyclicity of the trace show that

$$\mathrm{Tr}\left\{(-)^F\left\{Q_\alpha\ ,\ \bar{Q}_{\dot{\beta}}\right\}\right\} = 0\ .$$

- Then using $\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}}P_\mu$ show that

$$\mathrm{Tr}\left\{(-)^F\left\{Q_\alpha\ ,\ \bar{Q}_{\dot{\beta}}\right\}\right\} = 2\,(\sigma^\mu)_{\alpha\dot{\beta}}\,p_\mu\,\mathrm{Tr}\left\{(-)^F\right\}\ ,$$

Combining all the above and interpreting the trace as

$$\begin{aligned}\mathrm{Tr}\left\{(-)^F\right\} &= \sum_{\text{bosons}}\langle B|\,(-)^F\,|B\rangle\ +\ \sum_{\text{fermions}}\langle F|\,(-)^F\,|F\rangle \\ &= \sum_{\text{bosons}}\langle B|B\rangle\ -\ \sum_{\text{fermions}}\langle F|F\rangle = n_B\ -\ n_F = 0\ .\end{aligned}$$

4 SUSY algebra and Jacobi Identity

Prove that $\left[Q_\alpha\ ,\ P^\mu\right] = 0$ following the steps bellow:

- Assume that $\left[Q_\alpha\ ,\ P^\mu\right] = c\cdot(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{Q}^{\dot{\alpha}}$

(Is there an other way you can write an object with μ, α free indices that is linear in Q ?).

- To fix the constant c plug in the Jacobi identity for P^μ, P^ν and Q_α

$$\left[P^\mu\ ,\ \left[P^\nu\ ,\ Q_\alpha\right]\right]\ +\ \left[P^\nu\ ,\ \left[Q_\alpha\ ,\ P^\mu\right]\right]\ +\ \left[Q_\alpha\ ,\ \left[P^\mu\ ,\ P^\nu\right]\right] = 0 \tag{5}$$

5 $\mathcal{N} = 1$ SYM

Check that

$$\mathcal{L} = -\frac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{8\pi^2}\mathrm{tr}F_{\mu\nu}\tilde{F}^{\mu\nu} + i\mathrm{tr}\bar{\lambda}\bar{\sigma}^\mu D_\mu\lambda \tag{6}$$

is invariant under the transformations

$$\begin{aligned}\delta_\epsilon A_\mu &= i\bar{\epsilon}\bar{\sigma}_\mu\lambda - i\bar{\lambda}\bar{\sigma}_\mu\epsilon \\ \delta_\epsilon\lambda &= \sigma^{\mu\nu}F_{\mu\nu}\epsilon\end{aligned} \tag{7}$$

6 Quadratic divergencies cancel!

Using the help of either Quevedo (page 48) or Terning (page 52) calculate the one-loop self-energy corrections to the complex scalar of the chiral multiplet. Perform the calculation using the naive cutoff regularization. Observe that quadratic divergencies cancel. This is a general property of supersymmetric theories.