Introduction to the AdS/CFT correspondence
HW set 3

May 11, 2012

1 Reading

- Brand new review by Horatiu Nastase "Introduction to supergravity". arXiv:1112.3502
- "Supergravity", P. van Nieuwenhuizen Physics Reports Volume 68, Issue 4, February 1981, Pages 189–398
- "Supergravity, Brane Dynamics and String Duality", P. West hep-th/9811101
- "Supergravity" B. de Wit hep-th/0212245
- Polchinski "String theory" volume 2, chapter 12, pages 84-92.
- Green-Schwarz -Witten, "Superstring theory" volume 1 page 226 and volume 2, chapter 13.

2 Spinors in a gravitational background

Show that the action for a spinor (Dirac or Majorana) in a gravitational background

$$S_\psi \sim \int dx\, e\, \bar{\psi}\, \Gamma^\mu D_\mu \psi \tag{1}$$

is invariant under both general coordinate (Einstein) transformations

$$\begin{aligned} \delta_E e_\mu^a &= \xi^\nu \partial_\nu e_\mu^a + (\partial_\mu \xi^\nu) e_\nu^a \\ \delta_E \omega_\mu^{ab} &= \xi^\nu \partial_\nu \omega_\mu^{ab} + (\partial_\mu \xi^\nu) \omega_\nu^{ab} \\ \delta_E \psi_\mu &= \xi^\nu \partial_\nu \psi_\mu + (\partial_\mu \xi^\nu) \psi_\nu \end{aligned} \tag{2}$$

and local Lorentz transformations

$$\begin{aligned} \delta_{LL} \psi &= -\frac{1}{4} \lambda^{ab} \gamma_{ab} \psi \\ \delta_{LL} e_\mu^a &= \lambda^{ab} e_\mu^b \\ \delta_{LL} \omega_\mu^{ab} &= D_\mu \lambda^{ab} = \partial_\mu \lambda^{ab} + \omega_\mu^{ac} \lambda^{cb} - \omega_\mu^{bc} \lambda^{ca} \end{aligned} \tag{3}$$

3 Spin connection and Torsion

Begin by rewriting the Einstein-Hilbert action in terms of the vielbein e and the spin connection ω

$$S_{EH} = \frac{1}{16\pi G}\int d^d x\sqrt{-g}R = \frac{1}{16\pi G}\int d^4 x(\det e)R^{ab}_{\mu\nu}(\omega)e^{\mu}_{a}e^{\nu}_{b}\ . \tag{4}$$

Consider ω to be an independent field and derive it's equation of motion.[1] Show that this is precisely the "vielbein" postulate and implies Einstein-Hilbert gravity is torsion free

$$T^{a}_{[\mu\nu]} \equiv 2D_{[\mu}e^{a}_{\nu]} = 0\ . \tag{5}$$

Advice: you might find it usefull to first prove that

$$(\det e)e^{\mu}_{a}e^{\nu}_{b} = \epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}e^{c}_{\rho}e^{d}_{\sigma} \tag{6}$$

which follows from the definition of the determinant

$$\det e^{a}_{\mu} = \epsilon_{abcd}\epsilon^{\mu\nu\rho\sigma}e^{a}_{\mu}e^{b}_{\nu}e^{c}_{\rho}e^{d}_{\sigma}\ . \tag{7}$$

Then, add to it the Rarita-Schwinger action of a spin 3/2

$$S_{RS} = \frac{1}{2}\int dx\, e\,\bar{\psi}_{\mu}\,\Gamma^{\mu\nu\rho}D_{\nu}\psi^{\rho} \tag{8}$$

and calculate the torsion again.

4 $\mathcal{N} = 1$ supergravity in 3D

Start by counting the on-shell and off-shell degrees of freedom for $\mathcal{N} = 1$ supergravity in 3D. Show that the $\mathcal{N} = 1$ multiplet has no propagating degrees of freedom and that to go off-shell we need one bosonic auxiliary degree of freedom. In 3D $\gamma^{mnp} = -\epsilon^{mnp}$. Prove it and then use it to simplify the Rarita-Schwinger part of the supergravity action. Add an auxiliary scalar in the action. How should it transform under Einstein transformations and how under local Lorentz transformations? Check supersymmetry of the full Lagrangian using the 1.5 order formalism. Then compute $[\delta_1\,,\,\delta_2]\,e^{m}_{\mu}$. (For this exersise you might find useful to look at Nastase's chapter 6, or the "Lectures on supersymmetry and supergravity in (2+1)-dimensions" by F. Ruiz and P. van Nieuwenhuizen)

5 Type IIA supergravity in 10D

With the help of P. West hep-th/9811101 pages 58-66 (or Polchinski "String theory" volume 2, chapter 12, pages 84-89), derive the bosonic part of type IIA supergravity Lagrangian in 10D by dimensionally reducing $\mathcal{N} = 1$ supergravity in 11D.

[1] This is the so called first order formalism of gravity or Palatini formalism.