SPDE Gallery 2

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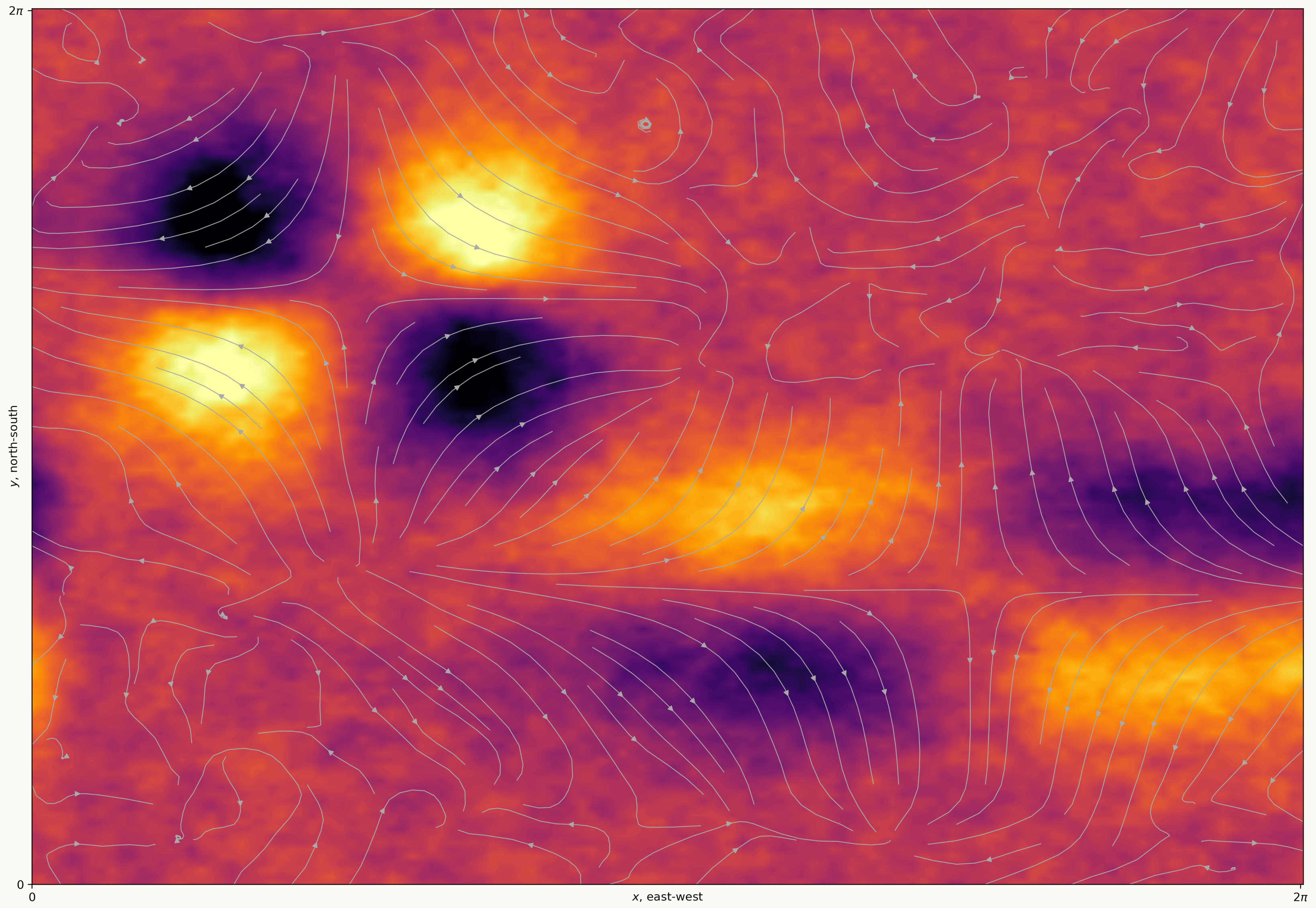
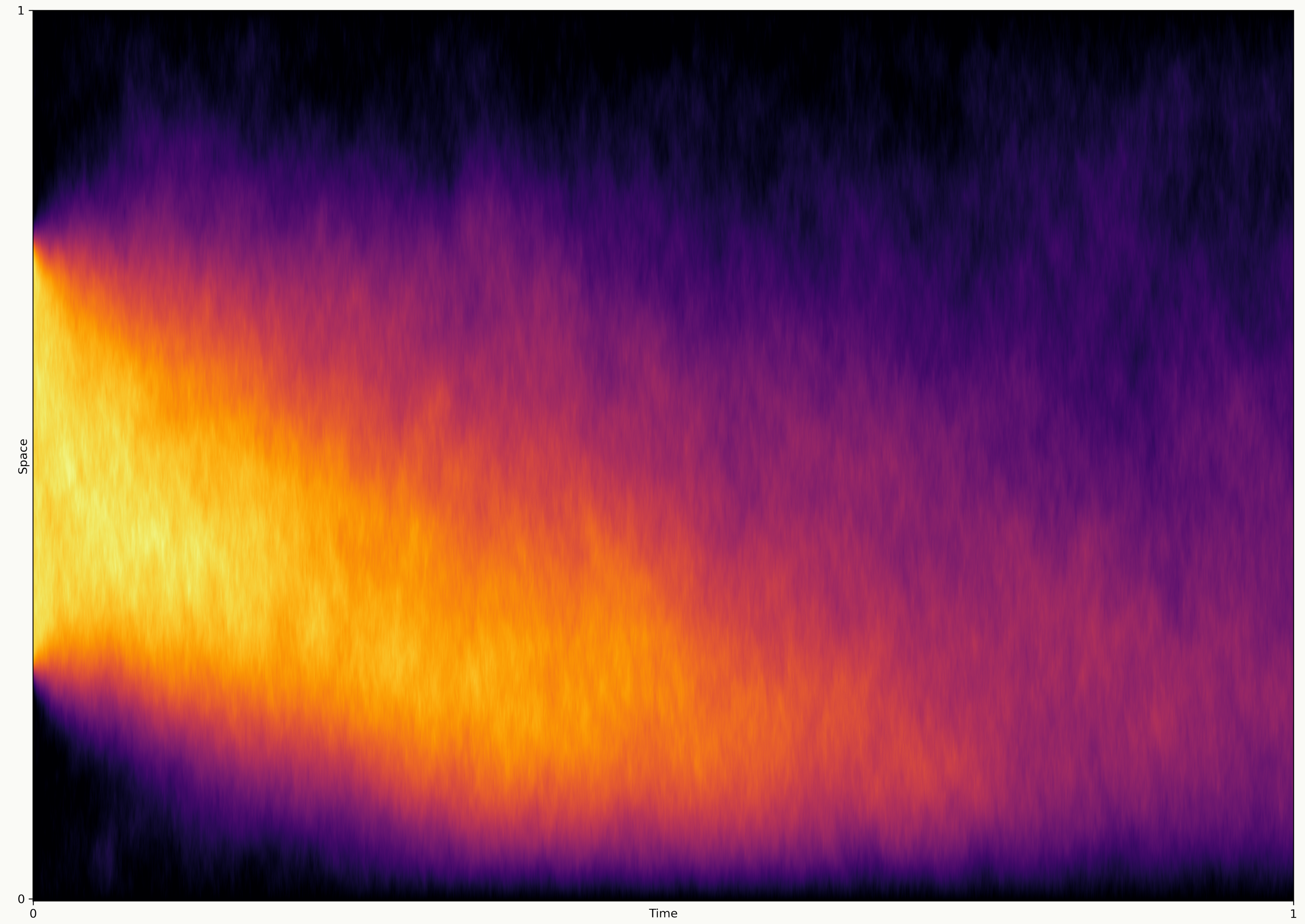


Incompressible 2D Navier-Stokes equations with stochastic forcing

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \nu\Delta\mathbf{v} + \boldsymbol{\sigma}\dot{W}(t,x,y), \quad \nabla\cdot\mathbf{v} = 0, \quad x,y \in [0,2\pi],\ t\in[0,2].$$

A reduced 2D model depicting the horizontal momentum equations in the 3D Primitive Equations (PEs), capturing the qualitative behavior of large-scale geophysical flows under the influence of nonlinear advection, viscous dissipation, and large-scale stochastic forcing. Estimation of the horizontal and vertical viscosities in stochastic PEs by the spectral method required a special decomposition of the solution in its barotropic and baroclinic components. Heatmap at terminal time $T = 2$ of the vorticity field $\omega = \partial_x v_2 - \partial_y v_1$, overlaid with streamlines of the velocity field.

Cialenco, I., Hu, R. & Lin, Q. (2023). Estimation of Anisotropic Viscosities for the Stochastic Primitive Equations. *Forthcoming in Stochastic and Partial Differential Equations: Analysis and Computations. arXiv preprint* arXiv:2309.06952.

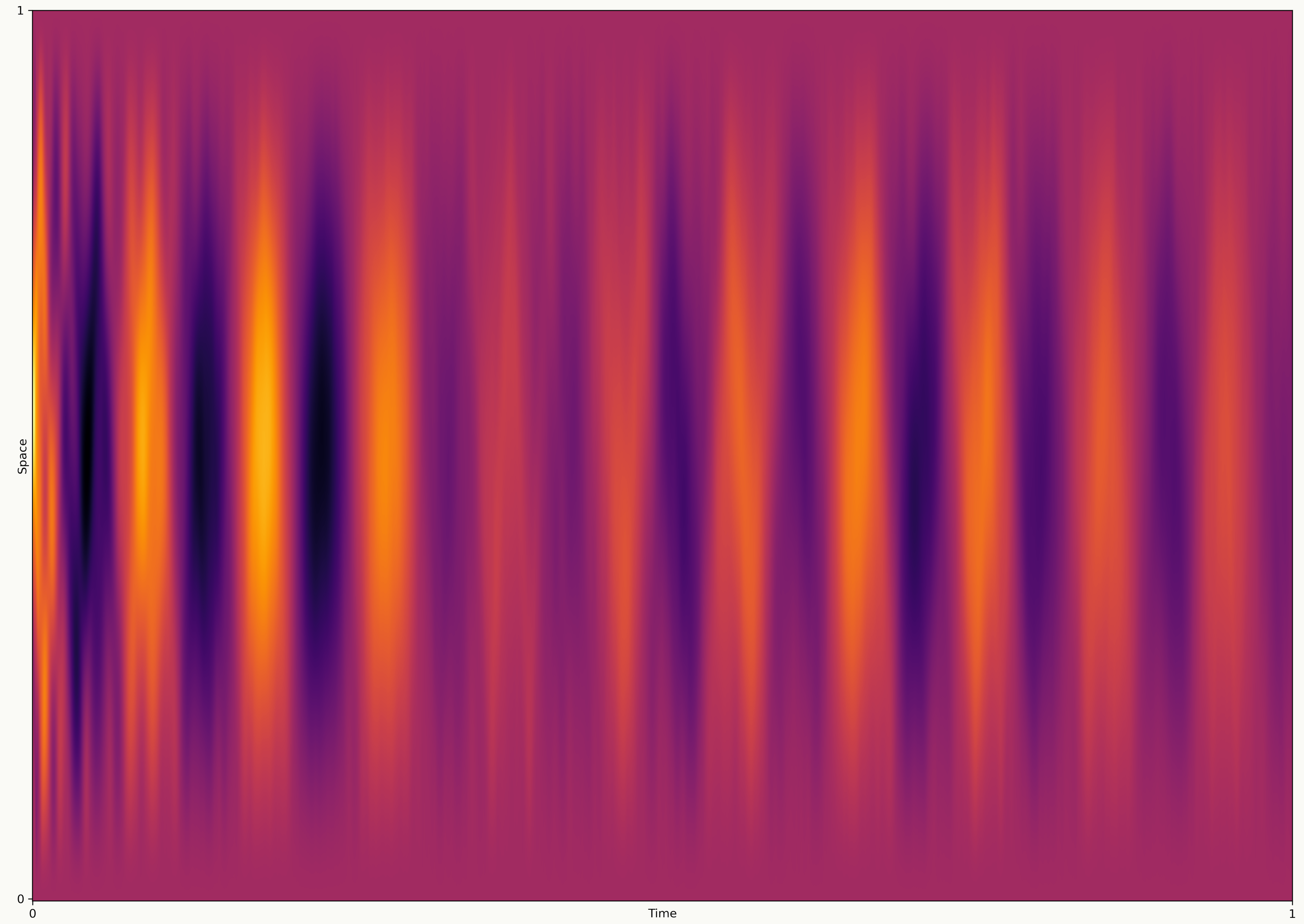


Stochastic Burgers equation driven by space-time white noise

$$\partial_t X(t,x) = \vartheta\partial^2_{xx}X(t,x) + \frac{1}{2}\partial_x\left(X^2(t,x)\right) + \sigma\dot{W}(t,x), \quad t,x\in[0,1].$$

The estimation of the viscosity parameter ϑ in the stochastic Burgers equation driven by space-time white noise can be performed as in the trace-class noise case, however it requires new regularity results for the solution and new arguments to prove the asymptotic normality of the estimator. Heatmap with $\vartheta = 0.05$ and $\sigma = 0.05$.

Janák, J. & Priola, E. (2025). Parameter estimation for the stochastic Burgers equation driven by white noise from local measurements. *arXiv preprint* arXiv:2503.09507.

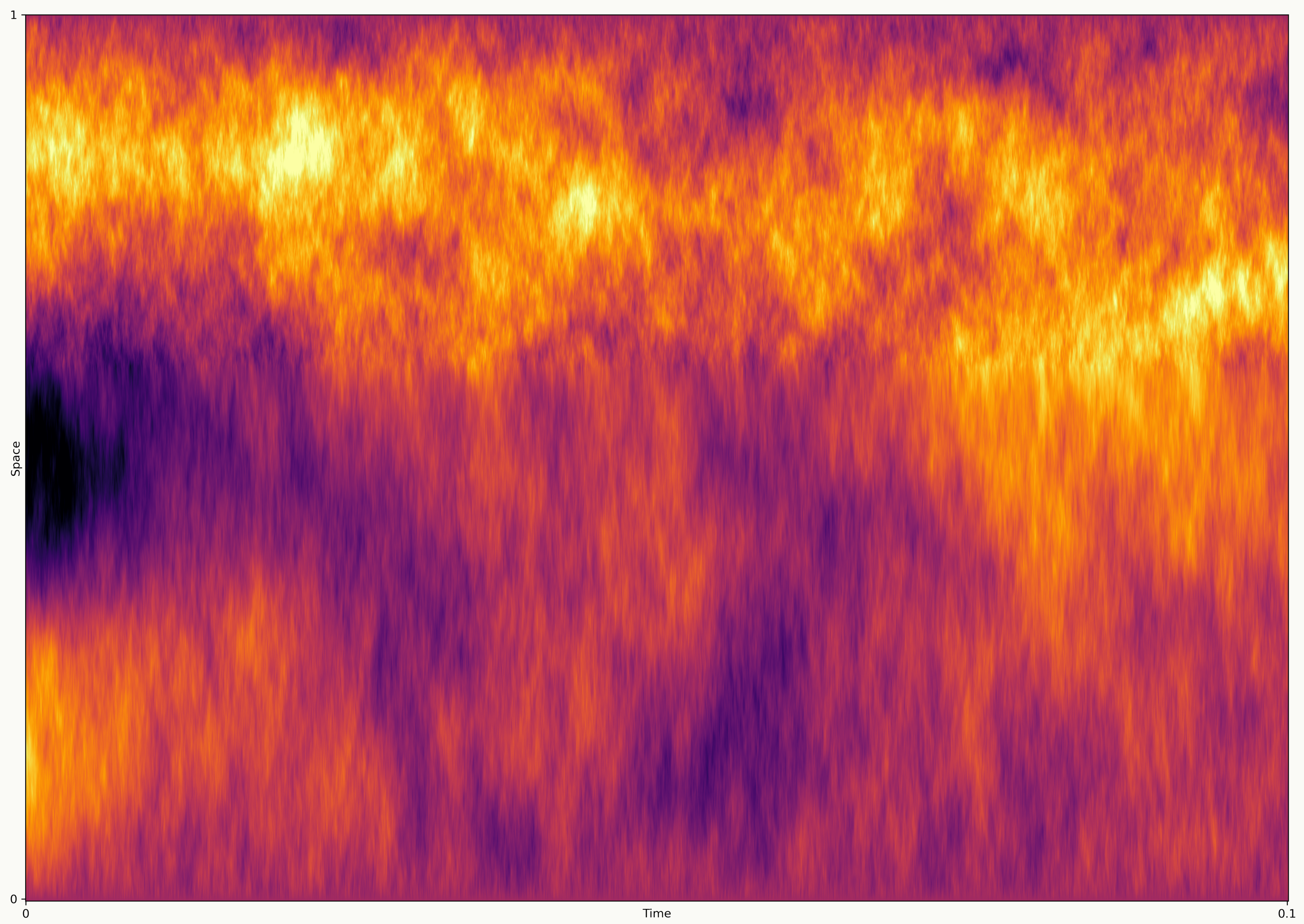


Stochastic (clamped) plate equation with damping

$$\partial^2_{tt}u(t,x) = \vartheta(\partial^2_{xx})^2u(t,x) + (\eta_1\partial^2_{xx} + \eta_2)\partial_t u(t,x) + \dot{W}(t,x), \quad t,x\in[0,1].$$

The elasticity parameter ϑ as well as weak and strong damping coefficients η_1 and η_2 can be jointly estimated from local measurements. The asymptotic analysis is intrinsically related to the solution theory of damped or amplified second-order Cauchy problems via M,N-functions. Heatmap with $\vartheta = 0.3$, $\eta_1 = 0.01$ and $\eta_2 = 0.3$.

Tiepner, A. & Ziebell, E. (2024). Parameter estimation in hyperbolic linear SPDEs from multiple measurements. *arXiv preprint* arXiv:2407.13461

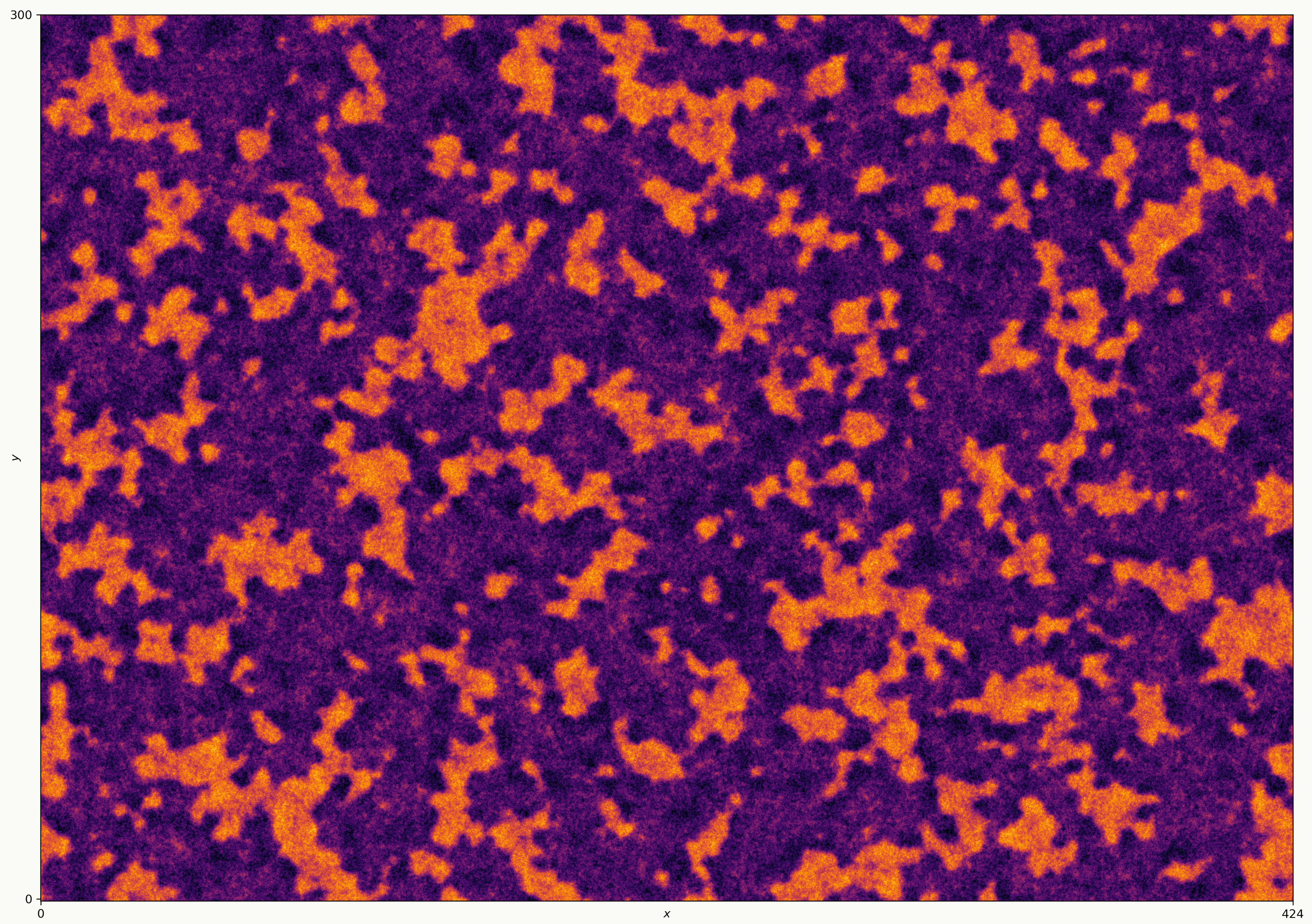


Stochastic heat equation with interface

$$\partial_t X(t,x) = \partial_x\theta(x)\partial_x X(t,x) + \dot{W}(t,x), \quad \text{with} \quad \theta(x) = \theta_-\mathbf{1}_{[0,\tau]}(x) + \theta_+\mathbf{1}_{(\tau,1]}(x), \quad x\in[0,1], t\in[0,0.1].$$

This is a model for stochastic heat flow with two materials of different conductivities θ_-, $\theta_+ > 0$. The statistical problem of estimating the interface point τ is non-regular and allows for faster estimation rates with non-Gaussian limit distributions. Under multiple local observations the analysis of a CUSUM-type M-estimator requires extensions of empirical process techniques, coupling ideas and fine regularity results for the semigroup around the interface τ. Heatmap with Dirichlet boundary conditions, $X(0,x) = \sin(3\pi x)$ and $\tau = 0.6$, $\theta_+ = 0.3$, $\theta_- = 1.0$.

Reiß, M., Strauch, C. & Trottner, L. (2023). Change point estimation for a stochastic heat equation. *arXiv preprint* arXiv:2307.10960.

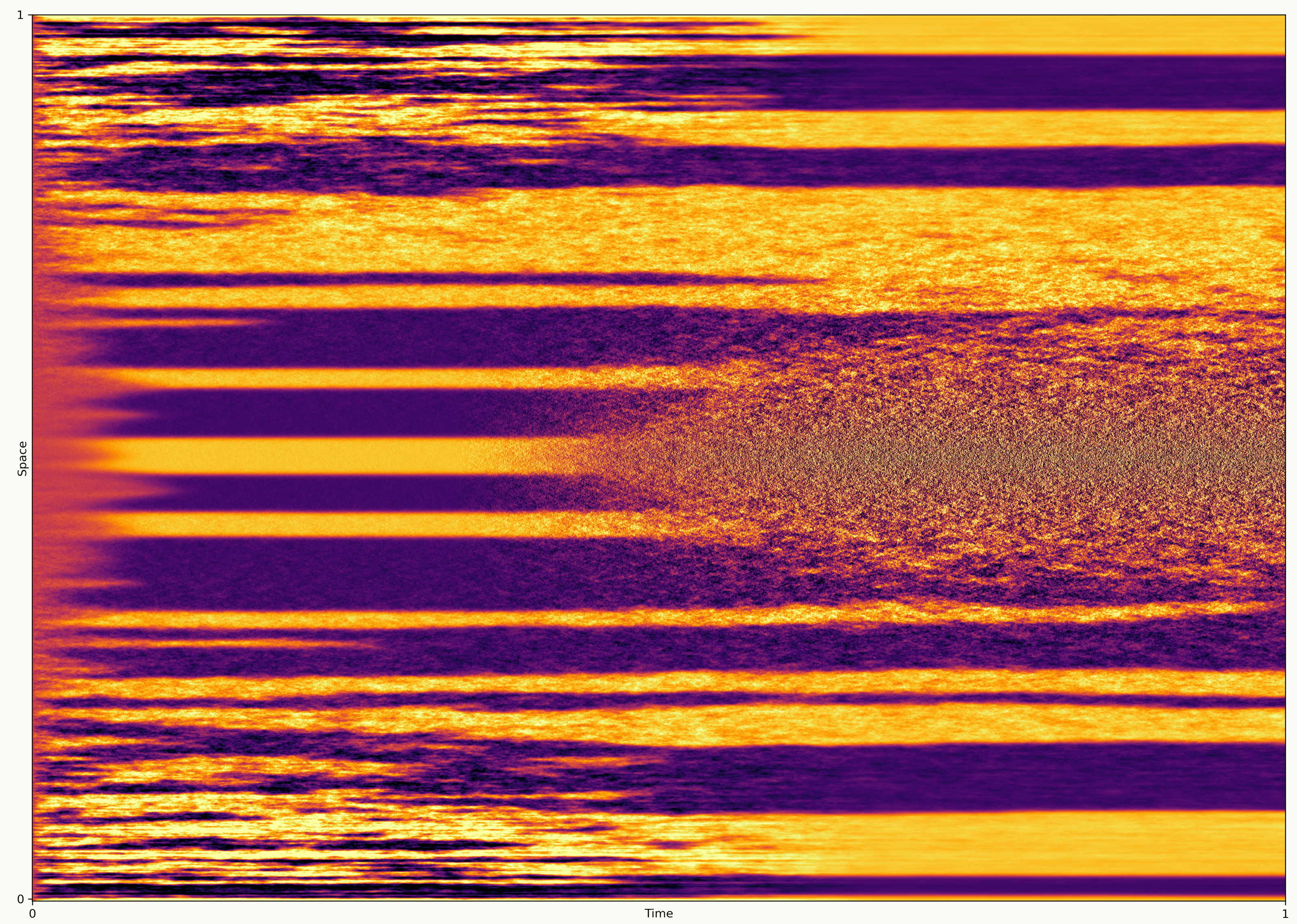


Activator-inhibitor system in two dimensions

$$\partial_t U(t,x,y) = D_U\Delta U(t,x,y) + U(t,x,y)\,(1 - U(t,x,y))\,(U(t,x,y) - 0.3) - V(t,x,y) + \sigma\dot{W}(t,x,y)$$
$$\partial_t V(t,x,y) = D_V\Delta V(t,x,y) + \varepsilon(bU(t,x,y) - V(t,x,y)), \quad t\in[0,1], \quad x\in[0,424], \quad y\in[0,300].$$

The simulation shows the activator component U of a stochastic FitzHugh-Nagumo system at time $T = 112$. The spots tend to form wave-like patterns, which is the result of the interplay between their tendency to spread out, and the action of the inhibitor, which dismantles the spots. The dark regions surrounding the spots indicate the presence of the inhibitor. This model is used to describe the actin concentration in D. discoideum giant cells. The parameters are chosen as $D_U = 0.1$, $D_V = 0.02$, $\epsilon = 0.02$, $b = 0.2$, $\sigma = 0.075$.

Pasemann, G., Flemming, S., Alonso, S., Beta, C., & Stannat, W. (2021). Diffusivity estimation for activator–inhibitor models: Theory and application to intracellular dynamics of the actin cytoskeleton. *Journal of nonlinear science*, 31(3), 59.



Stochastic reaction-diffusion equation with differently scaled fractional noise

$$\left(\partial_t - \nu\partial^2_{xx}\right)X(t,x) - \vartheta(X(t,x)) = \sigma(t,x)\dot{W}^{H(x)}(t,x), \quad t,x\in[0,1].$$

Scaling of fractional Gaussian noise depends heavily on the value of the Hurst parameter. The overall effect of the noise on the solution to a (parabolic) SPDE results from the interaction between the Hurst parameter H and the noise intensity σ, as shown in the heatmap. The importance of joint estimation of these two parameters motivates ongoing research on this topic. Heatmap with $\nu = 10^{-5}$, $H(x) = 2|x - 1/2|, x\in[0,1]$, $\vartheta(x) = 10x(1-x)(x-1/2)$ and $\sigma(x,t) = 0.06$ if $t < 1/3$, $\sigma(x,t) = (2-3t)0.06 + (3t-1)0.06(\Delta t)^{H(x)-1/2}$ if $1/3 < t < 2/3$ and $\sigma(x,t) = 0.06(\Delta t)^{H(x)-1/2}$ if $t > 2/3$.

Kříž P., Self-similarity and Hurst index estimation for parabolic SPDEs. Manuscript.

Poster arranged by Sascha Gaudlitz, Eric Ziebell & Markus Reiß. https://hu.berlin/SPDE-Gallery2