Übungen zur Analysis für Informatiker

Humboldt-Universität zu Berlin, Sommersemester 2017
PD Dr.habil. Olaf Müller
Exercise Sheet 2, due to 24th of April 2017, 12:00

Exercise 1: Minimum and maximum

Prove that a finite ordered set $(X, \geq)$ admits a minimum resp. a maximum, i.e., an element $x \in X$ such that $x \geq y$ resp. $y \geq x$ for all $y \in X$. Find a counterexample for the same statement without the condition of finiteness where X is

- a countable ordered field,
- a bounded set in an ordered field.

Exercise 2: Infimum property = Supremum property

Let us say that a an ordered field has the **infimum property** iff for every nonempty subset A of K, the set of lower bounds of K is either empty or contains a greatest element. Show that an ordered field has the infimum property if and only if it has the supremum property.

Exercise 3: Subsequences

Show that every sequence contains a monotonous subsequence. Find an example of a sequence containing a monotonously increasing subsequence and a monotonosly decreasing subsequence. Prove the following: If, for a sequence $x : \mathbb{N} \to \mathbb{Q}$, the sequence x_e of even members defined by $x_e(n) := x(2n)\ \forall n \in \mathbb{N}$ converges to a limit l_e, and if the sequence of odd members x_o defined by $x_o(n) := x(2n+1)\ \forall n \in \mathbb{N}$ converges to a limit l_o and if $l_e = l_o$ then the sequence x itself converges to $l_e = l_o$.

Exercise 4: Convergence and boundedness

Find examples of sequences in $\mathbb{Q}$ that are

- bounded, but not Cauchy;
- Cauchy, but not convergent.