Übungen zur Analysis für Informatiker

Humboldt-Universität zu Berlin, Sommersemester 2017
PD Dr.habil. Olaf Müller
Exercise Sheet 2, due to 24th of April 2017, 12:00

Exercise 1: Minimum and maximum

Prove that a finite ordered set $(X, \geq)$ admits a minimum resp. a maximum, i.e., an element $x \in X$ such that $x \geq y$ resp. $y \geq x$ for all $y \in X$. Find a counterexample for the same statement where X is

- a countable ordered field,
- a bounded set in an ordered field.

Exercise 2: Infimum property = Supremum property

Let us say that a an ordered field has the **infimum property** iff for every nonempty subset A of K, the set of lower bounds of K is either empty or contains a biggest element. Show that an ordered field has the infimum property if and only if it has the supremum property.

Exercise 3: Subsequences

Show that every sequence contains a monotonous subsequence. Find an example of a sequence containing a monotonously increasing subsequence and a monotonosly decreasing subsequence. Prove that if the sequence of even members converge and the sequence of odd members converge and the respective limits coincide then the sequence converges to the limit of the subsequences.

Exercise 4: Convergence and boundedness

Find examples of sequences in $\mathbb{Q}$ that are

- bounded, but not Cauchy;
- Cauchy, but not convergent.