Abstract: "Stochastic Volatility Asymptotics"

(Four lectures based on FPSS-2011 CUP book)

I will first present what do we mean by fast and slow processes characterized respectively by their time scales ϵ and $1/\delta$ for small parameters ϵ and δ. In particular, the fast time scale is intimately related to ergodicity or "mean-reversion". Examples (OU and CIR processes) will be discussed (based on chapter 3).

Then, using Feynman-Kac formula we characterize option prices under stochastic volatility as solutions of linear parabolic PDEs with terminal conditions. Asymptotics for this type of equations as $\epsilon \to 0$ and $\delta \to 0$ lead respectively to singular and regular perturbation methods which will be explained (based on Chapter 4).

By inverting the Black-Scholes formula (explicit formula for European call option prices under constant volatility), option prices are transformed into implied volatilities which play a central role in practice. We will see how our price expansions translate into implied volatility expansions used for model calibration to market data (based on chapter 5).

The same pricing problem can also be approached by martingale methods which are essential for hedging options (based on chapter 8).

The Merton problem of portfolio optimization under stochastic volatility leads to nonlinear HJB type PDEs. We will see that perturbation methods can also be applied in this nonlinear context (based on the FSZ-2013 paper).