

Prof. Peter Friz invites you to attend the Mini-Course

## „Geodesics, Laplacians and random walks in sub-Riemannian geometry“

presented by Professor **Ugo Boscain** at the Technische Universität Berlin, Institut für Mathematik, AG Stochastik und Finanzmathematik, Raum MA 748, Strasse des 17. Juni 136, 10623 Berlin.

### Professor Ugo Boscain (Paris, Polytechnique)

is one of the leading specialists in geometric control theory, a topic with many connections to optimization, partial differential equations, calculus of variations and probability theory. His recent research is supported by ERC.

In this series of Lecture **Ugo Boscain** will discuss properties of geodesics in sub-Riemannian geometry. He will prove a suitable version of the Pontryagin Maximum Principle and will discuss normal and abnormal extremals. He will then discuss how to define an intrinsic Laplacian in sub-Riemannian geometry as limit of geodesic random walks. The content of the course will be quite flexible depending on the audience.

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| <b>Lecture 1</b>   | <b>Monday, 18 Jan, 16:30 - 18:00</b> | <b>MA 748</b> |
| Definition of sub-Riemannian manifold. Carnot-Carathéodory distance. Rank-varying and non-equiregular structures. The Chow theorem. The Hopf-Rinow theorem. Hausdorff dimension. |                                      |               |

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| <b>Lecture 2</b>   | <b>Wednesday, 20 Jan, 9:30 - 11:00</b> | <b>MA 748</b> |
| First order conditions. Normal and abnormal extremals. Proof that normal extremals are always geodesics (i.e. such that small arcs are minimizers of the distance). Examples: Heisenberg, Grushin, Martinet. |  |               |

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| <b>Lecture 3</b>   | <b>Thursday, 21 Jan, 9:30 - 11:00</b> | <b>MA 748</b> |
| First order conditions. Normal and abnormal extremals. The problem of defining an intrinsic volume in SR geometry. The Popp volume, The Hausdorff volume. The horizontal gradient. The macroscopic Laplacian defined as divergence of the horizontal gradient. |                                       |               |

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| <b>Lecture 4</b>   | <b>Friday, 22 Jan, 9:30 - 11:00</b> | <b>MA 748</b> |
| The Laplacian defined as limit of geodesic random walks. The Ito and the Stratonovich random walks. Sampling the volume. The Riemannian case. The 3D contact case. Surprising results in higher dimensional contact manifolds. |                                     |               |

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