Minicourse: "Adaptive Dynamics"

In this series of lectures I will review some by now classical aspects of stochastic models for adaptive dynamics but also point to some recent and quite exciting developments that have been triggered by interactions with oncologists. Stochastic particle models for adaptive dynamics have been proposed by Diekmann and Law and further pushed by Ferrière, Méléard, Champagnat, and others.

The starting point is Markov process on a state space of point measures on some "trait space". This process models the basic genetic mechanisms of birth, death, mutation, and in particular, interaction/competition, where all the corresponding parameters depend on the individuals location in trait space. Mathematical results can be obtained due to the introduction of scaling parameters: the large population size, K, the small mutation rate, μ, and the small size of the mutation step d. Choosing these parameters in dependence of each other, and choosing appropriate time scales, one can derive deterministic or stochastic limit processes: Lottka-Volterra systems, the trait substitution sequence, the canonical equation of adaptive dynamics, and phenomena such as evolutionary branching.

A seemingly natural setting to apply these kind of models is tumour growth. Towards the end of the lecture I will report on some model extensions and that were motivated by a cooperation with oncologist from Bonn, and on some early attempts to implement these ideas in simulations for immunotherapy of melanomas.