

# An Introduction to Random Interlacements

RTG 1845 minicourse

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## Abstract

In these lectures we will give an introduction to the model of random interlacements that has been introduced by Sznitman [Szn10] in 2007.

This model has originally been motivated by questions about the disconnection of discrete cylinders and tori by the trace of simple random walk (as well as by related problems that have been investigated in the theoretical physics literature). Intuitively, random interlacements is a random subset of  $\mathbb{Z}^d$ ,  $d \geq 3$ , which appears as the limiting distribution of the trace of simple random walk on a large torus when it runs up to times proportional to the volume. It serves as a model for corrosion and in addition gives rise to interesting and challenging percolation problems.

The principal result imposing the structure of these lectures is the non-triviality of the percolation phase transition for the vacant set (i.e., the complement of random interlacements).

We will start with giving a short review of basic potential theory and then motivate how the model appears as the limiting distribution of simple random walk on the torus. Next, to obtain a better feeling for random interlacements we will compare it to the well-known model of Bernoulli percolation and, in particular, show that there is no domination of one of them by the other. We will then focus on the strong correlations of the model and show how these can be dealt with by the use of so-called “decoupling inequalities”. These can then be employed in order to deduce the main result.

If time admits we will sketch some more recent developments and applications of random interlacements, in particular with a view to the Gaussian free field.

The lectures will cover part of the material of the introductory text [DRS14], a preliminary version of which is available at

<http://www.math.columbia.edu/~drewitz/SpringerRI.pdf>

## Lectures:

- 1.) Tuesday 24.6., 16:15 – 17:45, TUB, MA748 (RTG Lounge)
- 2.) Thursday 26.6., 10:15 – 11:45, HU, 1.023 (BMS Lounge)
- 3.) Friday 27.6., 10:15 – 11:45, TUB, MA748 (RTG Lounge).

## References:

- [DRS14] Alexander Drewitz, Balázs Ráth, and Artem Sapozhnikov. *Lecture notes on random interlacements*. Springer Briefs in Mathematics. Springer, Heidelberg, 2014.
- [Szn10] Alain-Sol Sznitman. Vacant set of random interlacements and percolation. *Ann. of Math. (2)*, 171(3):2039–2087, 2010.