The main goal of this course is to provide an introduction to the theory of Backward Stochastic Differential Equations (BSDEs) and to its strong connection to Finance. We will survey the classical existence/uniqueness results for this equations together with some proofs making the course accessible to anyone who is not familiar with the theory of BSDEs, and we will present the link between BSDEs and several problems in Finance, like for instance the utility maximization problem. We will also consider new questions in this area like the existence and the estimation of densities for the marginal laws of the solution processes together with some new results for the Malliavin differentiability of BSDEs. For each of these questions, we will review the classical results and present some new developments. The course will be divided into four lectures and we provide below a brief description of them.

Lecture 1: In the first lecture, we will show how BSDEs naturally arise in several problems in Finance, like the utility maximization problem, the representation of risk measures, or in superhedging and quantile hedging problems.

Lecture 2: In the second lecture, we will provide general existence/uniqueness results for Lipschitz and quadratic BSDEs.

Lecture 3 and 4: Finally, we will deal with a theoretical issue which is motivated by the numerical analysis of quadratic BSDEs, that is, the existence of densities for the marginal laws of the solution to a BSDE. This problem has been very few studied and we will revisit and extend the results of the literature. Our approach is based on the use of the Malliavin calculus and of the Nourdin-Viens' formula that we will recall in this course. In particular, we will present a new method and new conditions to prove the Malliavin differentiability for the solution to a BSDE.