"The Φ^4 model: Renormalisation, particle approximations and global analysis"

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The Φ^4 model is a classical model in statistical mechanics. It arises naturally when trying to construct a version of the Ising model with continuous spins (rather than ± 1 valued spins) defined on continuous space (rather than a lattice). The construction of this model involves difficult issues such as working with random distributions rather than functions and the appearance of "infinite terms" that have to be removed. Over the last few years remarkable progress has been made in the understanding of such singular stochastic objects - most prominently by Hairer and Gubinelli - and in this series of lectures I aim to present some of the key ideas and their implications in the context of this very interesting model.

The first lecture will be introductory: I will explain in some detail the derivation of the Φ^4 model, basic scaling and regularity issues as well the concept of renormalisation. I will then show a perturbative approach which is at the heart of the solution theories by Hairer and Gubinelli. The main aim of the second lecture will be to discuss how the Φ^4 model arises as a scaling limit of Ising models with long range interaction. Finally, the third lecture I will show how to obtain bounds on behaviour of the system at large scales with PDE methods. The results presented in the second and third lecture are joint work with Jean-Christophe Mourrat.