Abstract: "Central Limit Theorem for additive functionals of some Markov processes: anomalous results"

In this talk we will consider an ergodic Markov process X_t ($t \in \mathbb{N}$ or $t \in \mathbb{R}^+$) with unique invariant probability μ, and some additive functional $S_t = \sum_{k=1}^{t} f(X_k)$ or $S_t = \int_0^t f(X_s)ds$ for some μ centered f.

If $f \in \mathbb{L}^2(\mu)$ the expected appropriate normalization is $\sqrt{\mathrm{Var}(S_t)}$ (expected to be of order $\sqrt{t}$), and the expected limit is then a standard gaussian. If $f \in \mathbb{L}^p(\mu)$ ($1 < p < 2$) one expects in some cases some stable limit after appropriate normalization. It turns out that the mixing rate of the process (equivalently the rate of convergence to equilibrium) is of particular importance for these results to hold true. We shall recall some of the main recent (and less recent) results in this direction and explain how the mixing rate enters into the game.

We shall also discuss a particular class of examples for which depending on whether the convergence to equilibrium is quick enough or not, anomalous limit (with some variance breaking) or anomalous normalization appear. At the level of the invariance principle instead of the simple CLT theorem, the expected limiting process becomes a fractional Brownian motion instead of the usual one. These examples correspond to a special class of kinetic P.D.E.'s with heavy tails equilibria.