Abstract: "A Stochastic Free Boundary Problem and Limit Order Book Model"

Free boundary problems allow for modeling of multi-phase systems with separating boundaries evolving in time. We want to model a stock market with a large amount of transactions in rather short time and consider buy- and sell-side of the limit order book as a price-time-continuous two-phase system. Doing so, we introduce an infinite dimensional model based on a generalized stochastic Stefan problem and analyze the resulting second order SPDE with free boundary.

In order to show existence of a solution we transform the problem into a stochastic evolution equation. The boundary interaction introduces a new drift term which has no chance to get controlled in any reasonable Banach space. However, the mild formulation on the domain of the Laplacian allows to use the smoothing effects of its semigroup. Despite of the non-standard setting for the stochastic evolution equation we show existence of an unique maximal mild solution of the general model; extending results of Kim, Sowers and Zheng. This solution can be shown to be continuous in the graph norm and to solve the stochastic equation even in analytically strong sense - up to the stopping time. However, additional assumptions on the boundary interaction lead to non-explosion and global existence. Finally, we see that a Nagumo type condition is sufficient for positivity, which should be a natural property of a limit order book model.