"Polynomial Chaos and Scaling Limits of Disordered Systems"

Francesco Caravenna, University of Milano-Bicocca

In this mini-course we consider statistical mechanics models defined on a lattice, subject to an external random field (called disorder). Examples of such "disordered systems" include the random field Ising model, directed polymers in random environment and disordered pinning models. Our goal is to study these models in a suitable continuum and weak disorder regime.

The key issue we address is the persistence of disorder in such a scaling limit. Focusing on the partition functions of the models, we will develop a unified approach, based on "polynomial chaos" expansion techniques, which is largely model independent and sheds light on the physical notion of "disorder relevance". Along the way, we will prove general results of independent interest, beyond statistical mechanics, such as a multi-linear extension of the Central Limit Theorem, based on a refined Lindeberg principle.

Special attention will be devoted to the directed polymer in random environment, which provides a link with the Stochastic Heat Equation (SHE) with multiplicative noise, where the notion of disorder relevance corresponds to the "sub-criticality" of the stochastic PDE. Of particular interest is the "critical" case, corresponding to the two-dimensional SHE, in which the standard approach breaks down. The last part of the course will be devoted to presenting recent progress on this issue, based on a fine multi-scale analysis of the structure of the noise.

(Based on joint work with R. Sun and N. Zygouras)

Some references:

-F. Caravenna, R. Sun, N. Zygouras. Polynomial chaos and scaling limits of disordered systems. J. Eur. Math. Soc. (JEMS), to appear.
-T. Alberts, K. Khanin, J. Quastel. The intermediate disorder regime for directed polymers in dimension 1 + 1. Ann. Probab. 42 (2014), 1212-1256.
-E. Mossel, R. O'Donnell, K. Oleszkiewicz. Noise stability of functions with low influences: Variance and optimality. Ann. Math. 171 (2010), 295-341.