"Metastability of interacting particle systems at low temperature"

Frank den Hollander, Leiden University, The Netherlands

In this mini-course we will be looking at systems of particles that are subjected to a stochastics dynamics at low temperature. When the parameters of the system are chosen close to a "first-order phase transition line", the system exhibits metastable behaviour, i.e., when started in a "metastable phase" that is not the "stable phase", the system stays for a very long time in a "quasi-equilibrium" until it crosses over to the "equilibrium". (Think of a vapor that is cooled down but needs time to condense to a liquid.) The crossover is triggered by the occurrence of a "critical droplet" of the stable phase inside the metastable phase. The goal will be to compute the crossover time and to identify the size and shape of the critical droplet.

The main technique that will be used to reach this goal is potential theory of Markov chains. We will recall the key ingredients of potential theory that are relevant for metastability, and use them to build up a general theory of metastability for lattice-based particle models at low temperature. The key tool is the notion of capacity for pairs of disjoint sets of configurations, and variational principles for these capacities.

Our task will be to establish the universal metastable behaviour of our models under a number of general hypotheses. These hypotheses will subsequently be verified for two specific choices of system and dynamics:

1. Ising spins with Glauber dynamics.
2. Lattice gas particles with Kawasaki dynamics.

The focus will be on Metropolis dynamics, i.e., dynamics driven by an interaction Hamiltonian representing the energy landscape on the set of particle configurations. We will briefly mention other dynamics as well.

At the end of the mini-course we discuss some key challenges, e.g. metastability for particle systems in the continuum.

Background knowledge: Basic probability theory, plus key elements of variational calculus and statistical physics.