"Applications of paracontrolled distributions"

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Outline:

This course is an introduction to recent developments in the application of paraproducts to the study of singular SPDEs arising from scaling limits of space-time random dynamics. No previous knowledge of the subject is assumed apart from some familiarity with parabolic equations in Hölder or Sobolev spaces and some acquaintance with Gaussian random variables (Wick's formula, chaos decomposition).

Some introductory material is provided in the lecture notes which I wrote together with Nicolas Perkowski. To be found at

- M. Gubinelli, N. Perkowski. Lectures on Singular Stochastic PDEs. http://arxiv.org/abs/1502.00157

A good standard reference on paraproducts and Besov spaces is

- Bahouri, Hajer, Jean-Yves Chemin, and Raphaël Danchin. 2011. Fourier Analysis and Nonlinear Partial Differential Equations. Springer.

and another one on Gaussian spaces is

- Janson, Svante. 1997. Gaussian Hilbert Spaces. Cambridge University Press.

I will try to cover applications of paraproducts and paracontrolled calculus to a series of models.

1) Hamiltonian/Parabolic Anderson model in 2 dimensions
2) The Kardar-Parisi-Zhang equation and the Stochastic Heat equation in 1 dimension
3) The Φ^4_3 model (or stochastic quantisation equation in 3 dimensions)

Outline of the lectures (5 sessions of 1h30)

1. Introduction to the models. Singular SPDEs as scaling limits of microscopic models. Description of the limiting dynamics via renormalised products. The general strategy.
2. Besov spaces and the decomposition of products. Paraproducts and paracontrolled distributions. Applications to some of the models.

3. The stochastic quantisation equation and renormalisation. The issue of global space-time solutions in 2d.

4. KPZ equation via paracontrolled distributions. Relations to the stochastic heat equation. Stochastic control interpretation. Global in time solutions and positivity of the solutions to the stochastic heat equation.