Summer School 2015
Berlin ∘ Padova

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00-10:30	**Caravenna**	Gubinelli	**Caravenna**	Gubinelli	**Caravenna**
10:30-11:00	Break	Break	Break	Break	Break
11:00-12:30	Gubinelli	**Caravenna**	Gubinelli	**Caravenna**	Gubinelli
12:30-14:30	Lunch	Lunch	Lunch	Lunch	Lunch
14:30-15:00	Martin	Milanesi	Hike Piz De Levico	Krüger	
15:00-15:30	Orrieri	Altmayer		Andreis	
15:30-16:00	Chorowski	Zanella		Cannizzaro	
16:00-16:30	Break	Break		Break	
16:30-17:00	Mariucci	Mazzonetto		Coghi	
17:00-17:30	Bilarev	Calvia		Flegel	
17:30-18:00	Pigato	Prömel		Tovazzi	
18:00-18:30		Cordoni		Board meeting	
19:00	Dinner	Dinner	Dinner	Dinner	tba

Speakers and Abstracts of afternoon's talks

Randolf Altmayer (HU Berlin)

Title: Estimating occupation time of continuous semimartingales (joint work with Jakub Chorowski)

Abstract: Let X be a continuous semimartingale. Given discrete data $\{X_{k/n}, k = 0,\ldots,n\}$ we study the estimation of the occupation time $\int_0^t f(X_r)\,dr$ for suitable functions f. Our main results concern the rates of convergence of the Riemann-type estimator $n^{-1}\sum_{k=1}^{\lfloor nt\rfloor} f(X_{\frac{k-1}{n}})$. For various smoothness assumptions on f, we are able to improve and simplify previous results, which are limited to diffusions. In particular, our results allow us to explain well-known but so far unintuitive rates in the case that f is an indicator function, namely by exact decay assumptions on the Fourier transform of f. Finally, in many cases we are able to provide central limit theorems which can be used for statistical inference.

Luisa Andreis (Università di Padova)

Title. McKean-Vlasov limit for interacting systems with simultaneous jumps

Abstract. We consider systems of N weakly interacting diffusions with jumps, having the peculiar feature that the jump of one component may induce simultaneous jumps of all others. Models belonging to this class have been proposed for the dynamics of neuronal systems and their limiting ($N \to +\infty$) behavior has been studied only for special cases in the positive half-line. In this talk, after briefly presenting our approach to prove propagation of chaos and derive the corresponding McKean-Vlasov equation for globally Lipschitz coefficients, we focus on some interesting cases where global Lipschitzianity fails. In particular we consider systems of equations with jumps where the jump rate has a local Lipschitz behavior, generalizing existing models for neuronal networks. This is a joint work with Paolo Dai Pra and Markus Fischer.

Todor Bilarev (RTG, HU Berlin)

Title: Multiplicative Market Impact Model with Transient Impact: Stability, Absence of Arbitrage and Pricing of Options

Abstract: In this talk, we will discuss a market model with a single risky asset and a large trader whose actions can affect its price. Her proceeds from trading are uniquely defined for continuous strategies of bounded variation. We extend the model to more general (càdlàg) trading strategies by continuity considerations. In particular, the cost of a block trade is derived as a limit of continuous trades over shorter and shorter time interval. To obtain jumps in the limit of continuous processes, a suitable topology on the space of càdlàg paths is considered - the Skorokhod M1 topology. The proceeds process is shown to be continuous (on the input strategy) with respect to this topology; moreover, continuity w.r.t. the standard Skorokhod J1 topology and the uniform topology is also shown. The model can be linked to a Marcus canonical equation for which a Wong-Zakai approximation results hold. It will be shown that the market model is absence of arbitrage. Having specified our model for a very general class of trading strategies, we consider the problem of pricing European options by superreplication. In special co- ordinates, a (geometric) dynamic programming principle holds that could be applied to show that the minimal superreplication price is a viscosity solution of a non-linear PDE with gradient constraints. When the PDE admits a sufficiently regular solution, a

replicating strategy can be constructed.

Alessandro Calvia (Università di Milano Bicocca)

Title. Filtering of time-homogeneous pure jump Markov processes with noise-free observation

Abstract. In this talk we address a particular stochastic filtering problem, mainly characterised by observations not corrupted by noise. Let X and Y be a given couple of stochastic processes, with values in two Borel spaces $(I, \mathcal{I})$ and $(O, \mathcal{O})$ respectively. The unobserved (or signal) process X is a time-homogeneous pure jump Markov process, whose rate transition measure is known. The observed process Y is defined as $Y_t = h(X_t)$, $t \geq 0$, where $h\colon I \to O$ is a known surjective and measurable function. The aim is to provide an explicit equation for the filtering process $\pi_t(\varphi) = \mathbb{E}\left[\varphi(X_t) \mid \mathcal{Y}_t\right]$, where $\varphi\colon I \to \mathbb{R}$ is a bounded measurable function and $\mathcal{Y}_t = \sigma(Y_s\colon 0 \leq s \leq t)$ is the natural filtration of Y. The problem is tackled with the aid of known results from marked point processes theory and a martingale approach. We also write down an SDE for the conditional probability measure π_t and show that the initial value problem is well-posed. Finally, we give an example where an explicit solution to the filtering equation can be found and we illustrate some possible developments of the present work, including the associated control problem with partial observation. This talk is based on a joint work with Fulvia Confortola and Marco Fuhrman.

Giuseppe Cannizzaro (RTG, TU Berlin)

Title: The Polymer Measure with white noise potential

Abstract: As an application of the approach developed by Gubinelli, Imkeller and Perkowski in their, by now, celebrated paper "Paracontrolled Distributions and Singular SPDEs", we construct the Polymer measure with white noise potential in dimension $d = 2, 3$. Moreover, we show how, thanks to the new techniques, it is possible to prove intrinsic properties of such a measure such as his singularity with respect to the Wiener one. This is a joint work with Dr. Khalil Chouk.

Jakub Chorowski (*Humboldt-Universität zu Berlin, chorowsj@math.hu-berlin.de*)

Title: Scale invariant volatility estimation in scalar diffusions (based on arXiv: 1507.07139)

Keywords: *Diffusion processes; nonparametric estimation; spectral approximation; sampling frequency*:

Abstract: Consider a time-homogeneous Itô diffusion process X on a state space $S \subset \mathbf{R}$ with local dynamics governed by a drift function b and volatility σ. For a given frequency $\Delta^{-1} > 0$ and sample size $N > 0$ we observe the process X at

equidistant times $\Delta, 2\Delta, ..., N\Delta$. We want to, under smoothness constraints on the coefficients, propose an optimal identification method of the function σ.

The problem of nonparametric inference of diffusion coefficients was a very active field of research and is nowadays well established. Optimal estimation methods are different in different frequency regimes. In the high frequency case [1] proposed a consistent and asymptotically normal volatility estimator based on the properties of small time increments. In the low frequency regime [2] analysed the spectral estimation method based on an eigenpair of the transition operator of the diffusion X.

In this talk we want to analyse the performance of a locally integrated version of the spectral estimator constructed in [2], when applied to high frequency data. We will argue that, on a high probability event, spectral estimator becomes close to the volatility estimator from [1] and attains the minimax optimal (for Lipschitz coefficients) mean L^1 error rate.

References

[1] Florens-Zmirou, D. (1993) *On estimating the diffusion coefficient from discrete observations*, J. Appl. Prob 30, 790–804.
[2] Gobet, E., Hoffmann, M. ,Reiß, M. (2004) *Nonparametric estimation of scalar diffusions based on low frequency data*, The Annals of Statistics 32, 2223–2253.

Michele Coghi (Scuola Normale Superiore di Pisa)
Title. Propagation of chaos for interacting particles subject to environmental noise
Abstract. A system of interacting particles described by stochastic differential equations is considered. Opposite to the usual model, where the noise perturbations acting on different particles are independent, here the particles are subject to the same space-dependent noise. We prove a result of propagation of chaos and show that the limit PDE is stochastic and of inviscid type

Francesco Cordoni (Università di Trento)
Title. Stochastic functional delay differential equations with jumps and applications to option pricing
Abstract. We investigate certain aspects of a stochastic process X whose evolution depends on both its past history and its present state. In particular we consider a given stochastic functional delay differential equation (SFDDE) with jumps taking values in $\mathbb{R}^d$. Exploiting the notion of *segment at time t* of the process X, i.e. the path of the process X in the interval $[t-r, t]$, where r is a positive constant representing the length of the memory, we lift the aforementioned SFDDE in a suitable infinite dimensional space of Lebesgue square integrable function L^2,

showing that its solution is an L^2-valued Markov process whose uniqueness can be shown under standard assumptions of locally Lipschitzianity and linear growth for the coefficients. Moreover, taking into account a coupled forward-backward system and under mild assumptions on differentiability of the coefficients, we provide a Feynman-Kac type representation theorem. Eventually, previous results will be applied to the option pricing problem for path-dependent claims.

Franziska Flegel (WIAS, Berlin)

Title: Localization of the first Dirichlet-eigenvector in the heavy-tailed random conductance model

Abstract: Our goal is to describe the almost-sure behavior of a random walk among random conductances that is killed when it reaches the boundary of a centered, large but finite box. Especially, we want to determine the almost-sure asymptotics of the behavior when the box size grows to infinity. We assume that the walker moves with variable speed and that the conductances are i.i.d., positive, symmetric and bounded from above. However, they may be arbitrarily close to zero. Our two main question are: What is the time-dependent probability that the random walker is still alive? What is the best strategy to stay alive as long as possible? The answers to these questions are ultimately linked to the spectral gap and the principal eigenvector of the random walk generator with zero Dirichlet boundary conditions outside the box. For the sake of simplicity, we restrict ourselves to the special case where the conductances have a polynomial tail near zero. It turns out that there is a critical exponent of the polynomial tail. Above this exponent, the spectral gap exhibits the typical diffusive scaling. Below this exponent the spectral gap scales differently and we can show that the principal eigenvector localizes. This implies that for sufficiently heavy tails of the conductances near zero, the best survival strategy for the random walker is to find a deep trap inside the box and stay there for a very long time.

Jennifer Krüger (RTG, TU Berlin)

Title: The stochastic neural field equation with discontinuous firing rate

Abstract: In this talk we will discuss the existence and uniqueness of solutions to the stochastic neural field equation with Heaviside firing rate modeling the spatiotemporal evolution of neural activity in thin layers of cortical tissue. This equation can be categorized as a nonlocal (stochastic) differential equation with discontinuous nonlinearity. Using a fixed-point iteration we will construct mild solutions of the deterministic as well as of the stochastic field equation and will then discuss suitable criteria which allow us to obtain uniqueness.

Sara Mazzonetto (RTG, Potsdam Universität)

Title: Explicit representation and simulation of the transition densities of a Skew Brownian motion with two semipermeable barriers.

Abstract: In this talk an explicit representation of the transition densities of real-valued Brownian dynamics undergoing their motion through semipermeable barriers will be presented. The technique we used requires a fine analysis of the semigroup. Finally a generalised rejection sampling method for sampling such densities will be proposed.

Ester Mariucci (LJK Grenoble)

Title. Asymptotic equivalence of diffusion processes and its Euler scheme: Small variance case

Abstract. When looking for asymptotic results for some statistical model it is often useful to dispose of a global asymptotic equivalence, in the Le Cam sense, in order to be allowed to work in a simpler model. In this talk, after giving an introduction to the main characters involved in the Le Cam theory, I will focus on equivalence results for diffusion models. More precisely, I will discuss the global asymptotic equivalence between scalar diffusion models with unknown drift function and small variance on the one side, and nonparametric autoregressive models on the other side. The time horizon T is kept fixed and both the cases of discrete and continuous observation of the path are treated. The asymptotic equivalences are established by constructing explicit Markov kernels that can be used to reproduce one experiment from the other.

Jörg Martin (RTG, HU Berlin)

Title: A discrete approximation for PAM in $\mathbb{R}^2$

Abstract: We study the convergence of the solutions of the parabolic Anderson model on epsilon $\mathbb{Z}^2$ for vanishing epsilon by using a paracontrolled approach on discrete, weighted Besov spaces.

Paolo Milanesi (Université de Marseille)

Title. Symmetries of the Ising model's partition function in the fermionic representation

Abstract. We consider the two-dimensional Ising model with periodic boundary condition in the absence of an external field. It's known that this model maps exactly into a free lattice fermionic theory, characterized by anticommuting Grassmann variables. With the help of this powerful formalism we will show how to express the partition function of the Ising model as a suitable linear combination of gaussian Grassmann integrals. Then we exploit the Kramers-Wannier duality to discuss the behaviour of the periodic-periodic contribution to the partition function, by showing that it vanishes at the critical point.

Carlo Orrieri (Università di Pavia)
Title. Necessary conditions for optimal ergodic stochastic control problems
Abstract. The talk is based on the Pontryagin stochastic maximum principle as a tool to obtain necessary conditions for optimal control problems. In particular I will introduce a version of the theorem for the ergodic control of infinite dimensional stochastic systems. This is a work in progress with G. Tessitore and P. Veverka.

Paolo Pigato (Università di Padova)
Title. Density and tube estimates for hypoelliptic diffusion processes
Abstract. We consider hypoelliptic diffusions, under both strong and weak Hormander condition. Using Malliavin Calculus techniques we find Gaussian estimates for the density of the law of the solution in short time. We then apply these density estimates to show exponential two-sided bounds for the probability that the diffusion remains in a small tube around a deterministic path up to a given time.

David Prömel (RTG, HU Berlin)

Title: Rough differential equations on Besov spaces

Abstract: Abstract: Rough differential equations are solved for signals in general Besov spaces unifying in particular the known results in Hölder and p-variation topology. To this end the paracontrolled distribution approach, which has been introduced by Gubinelli, Imkeller and Perkowski to analyze singular stochastic PDEs, is extended from Hölder to Besov spaces. As an application we solve stochastic differential equations driven by random functions in Besov spaces and Gaussian processes in a pathwise sense.

Daniele Tovazzi (Università di Padova)
Title. Critical fluctuations in presence of different kinds of bifurcation
Abstract. In this talk, we give an overview on the dynamics of critical fluctuations in presence of different kinds of critical points. We consider some versions of two famous interacting particle systems: Curie-Weiss model and Kuramoto model. All systems, evolving on $[0, T]$, are subject to a mean-field interaction and, as the number of particles N goes to infinity, their limiting dynamics become deterministic and exhibit phase transition. While fluctuations of order $\sqrt{N}$ around these deterministic limits converge to a Gaussian processes for all regimes, if we also rescale time in such a way T goes to infinity as N does, various features arise depending on the regime (subcritical, critial, supercritical). We are interested in study fluctuations at critical points, as they show peculiar space-time scaling and their limits may not be Gaussian. In our presentation, we will present these *critical fluctuations* for models described above. As different models will exhibit different bifurcation points, we will point out how critical fluctuations modify according to

various kinds of bifurcations.
This is a joint work with Francesca Collet and Paolo Dai Pra.

Margherita Zanella (Università di Pavia)

Title. Existence and regularity of the density for solutions of stochastic differential equations with boundary noise

Abstract. Using the Malliavin calculus, we study the existence and regularity of densities for the solution of a nonlinear heat diffusion with stochastic perturbation of Brownian and fractional Brownian motion type. The talk is based on joint work with Stefano Bonaccorsi