

# Summer School 2013

## Zurich-Berlin

### Program

	Monday CAB	Tuesday CAB	Wednesday CAB
09.00-10.30	Nizar Touzi	Nizar Touzi	Nizar Touzi
10.30-11.00	Coffee Break		
11.00-12.30	Martin Hairer	Martin Hairer	Martin Hairer
13.00	Lunch CAB		
14.30-15.00	Sonja Cox	Kai Du	Social program
15.00-15.30	Moritz Voss	Blanka Horvath	
15.30-16.00	Matti Leimbach	Eva Lang	
16.00-16.30	Coffee Break		
16.30-17.00	Joscha Diel	Sebastian Riedel	
17.00-17.30	Maite Wilke Berenguer	Adrián Gonzales Casanova	
19.00			Conference Dinner

	Thursday CAB	Friday CAB
9.00-10.30	Nizar Touzi	Martin Hairer
10.30-11.00	Coffee Break	
11.00-11.30	Martin Hairer	Alberto Chiarini
11.30-12.00		Pierre-François Rodriguez
12.00-12.30		Erich Baur
13.00	Lunch CAB	
14.30-15.00	Atul Shekhar	Benjamin Gess
15.00-15.30	Marvin Müller	David Prömel
15.30-16.00	Alexandros Saplaouras	Alessandra Cipriani
16.00-16.30	Coffee Break	
16.30-17.00	Nicoletta Gabrielli	
17.00-17.30	Tim Kutzker	

The lectures in cyan are students' presentations.

# Speakers and abstracts of the students' lectures

## Erich Baur (University of Zurich)

Title: Spatial behavior of certain (an)isotropic random walks in random environment

Abstract: I will discuss exit laws from large balls of random walks in an i.i.d. random environment in  $\mathbb{Z}^d$ ,  $d \geq 3$ , which is close to the fixed environment corresponding to simple random walk. Under a symmetry condition on the measure governing the environment, the exit laws approach those of a symmetric random walk, whose underlying nearest-neighbor transition kernel is a small perturbation of the simple random walk kernel charging each neighbor with mass  $1/(2d)$ . This result is an extension of the work of Bolthausen and Zeitouni (2007) and my PhD thesis, where exit measures of isotropic perturbative random walks in an i.i.d. random environment are studied. Joint work with Erwin Bolthausen.

## Alessandra Cipriani (University of Zurich)

Title: Thick points for a Gaussian Free Field in 4 dimensions

Abstract: This joint work with Rajat Subhra Hazra is concerned with the study of the fractal dimension of thick points for a 4-dimensional Gaussian Free Field. We adopt the definition of Gaussian Free Field on  $\mathbb{R}^4$  introduced by Chen and Jakobson (2012) viewed as an abstract Wiener space with underlying Hilbert space  $H^2(\mathbb{R}^4)$ . We can prove that for  $0 \leq a \leq 4$  the Hausdorff dimension of the set of  $a$ -high points is  $4 - a$ .

## Alberto Chiarini (TU Berlin)

Title: Invariance Principle for Diffusions in Random Environment

Abstract: Diffusions in heterogeneous media (media with impurities) can frequently be described by its effective behaviour. This means that there is a homogeneous medium, the effective medium, whose diffusive properties are close to those of the real inhomogeneous medium when measured on long space-time scales. A process of averaging or homogenization takes place so that the complicated small scale structure of the material is replaced by an asymptotically equivalent homogeneous structure. We shall discuss the classical results by Papanicolau-Varadhan, Kozlov and others which concern homogenization of diffusion processes in a stationary and ergodic environment, which also include periodic and almost periodic coefficients as special cases.

## Sonja Cox (ETH Zurich)

Title: Regularity and convergence rates for SDEs with non-globally Lipschitz coefficients

Abstract: In joint work with Martin Hutzenthaler and Arnulf Jentzen we study the regularity with respect to the initial data of SDEs with non-globally Lipschitz continuous coefficients. More precisely, we provide conditions under which the solution process depends on the initial data in a Lipschitz-continuous fashion. I will explain these results and provide examples of SDEs

with non-globally Lipschitz continuous coefficients that satisfy our conditions. Ultimately, we hope to use this to prove convergence rates of approximation schemes for such SDEs.

### **Joscha Diel (postdoc - TU Berlin)**

Title: Rotation invariants of 2D curves and applications to character recognition

Abstract: I introduce a new class of rotation invariants of curves in two dimensional space. These invariants are based on its iterated integrals. The invariants I present are complete in a sense and I describe an algorithm to calculate them. An application to online (stroke-trajectory based) character recognition is given. This seems to be the first time in the literature that the use of the iterated integrals of a curve is proposed for machine learning applications.

### **Kai Du (ETH Zurich)**

Title: Necessary conditions for optimal stochastic control in infinite dimensions

Abstract: Recently some progress has been made on an unsolved problem in control theory, i.e., the general stochastic maximum principle for infinite-dimensional stochastic control systems. In contrast to the finite-dimensional case, the main difficulties here lie in the second-order duality analysis and the characterisation of the second-order auxiliary process. In this talk I will introduce some related results and present our solution to this problem. The talk is partially based on a joint work with Dr. Qingxin Meng.

### **Nicoletta Gabrielli (ETH Zurich)**

Title: Pathspace Representation of Affine Processes

Abstract: A common problem faced in mathematical finance is the efficient computation of expectations of functionals arising from the pricing of derivative contracts. A possible way to look at this quantity is by means of the Kolmogorov equation corresponding to the pricing problem. One of the main features of affine-type operators is its degeneracy and the lack of Lipschitz regularity. In this talk we analyze a new representation of affine processes as path-space valued Lévy processes. This new representation not only leads to a new perspective on numerics of affine processes but is also essential to prove regularity of degenerate Kolmogorov equations with unbounded initial condition.

### **Benjamin Gess (HU Berlin)**

Title: Finite speed of propagation for stochastic porous media equations

Abstract: We prove finite speed of propagation for stochastic porous media equations perturbed by linear multiplicative space-time rough signals. Explicit and optimal estimates for the speed of propagation are given. The estimates are then used to prove that the corresponding random attractor has infinite fractal dimension.

## **Adrián Gonzales Casanova (RTG TU Berlin)**

Title: Seed banks, mixing times and convergence to the Kingman coalescent.

Abstract: In this talk we will explain some results on the long range effects in models of population genetics. We will describe a wide class of models that, under suitable rescaling, converges to the Kingman coalescent. Finally we generalize the proof of the convergence result, to provide an easy to verify criteria for structured coalescent models to converge to the Kingman coalescent.

## **Blanka Horvath (ETH Zurich)**

Title: The SABR Model from a functional analytic viewpoint

Abstract: Starting from the infinitesimal generator of the stochastic differential equation known as the SABR model of stochastic volatility, we present the geometric view on the SABR model introduced by Hagan, Lesniewski and Woodward. Our interest lies in an asymptotic formula for the probability density of the SABR process. We address the issue of the behavior of the process as the forward rate approaches zero and the shortcomings of the geometric viewpoint at zero forward. We introduce a functional analytic framework which allows for certain degeneracies of the infinitesimal generator. In this setting we propose a distance function, generalizing the Riemannian distance, which is better suited to describe short-time asymptotics of the probability density in the presence of degeneracies.

## **Tim Kutzker (WIAS Berlin)**

Title: Semi-Convergence in Distribution of Random Closed Sets in Topological Spaces

Abstract: We study the semi-convergence in distribution of random closed sets. Semi-convergence in distribution is described via convergence in distribution of random variables with values in a suitable topological space. We prove some convergence statements and finally for a sequence of random closed sets we give sufficient conditions for inner approximation in distribution to a random closed set. Note throughout the thesis we only assume a locally compact second countable Hausdorff-space.

## **Eva Lang (RTG TU Berlin)**

Title: Traveling Waves in Stochastic Neural Field Equations

Abstract: Many neurobiological phenomena are related to spatially structured states of activity in neural tissue. Trying to understand the dynamics underlying the formation of the observed activity patterns, the propagation of activity in a neural network consisting of a large number of neurons with nonlocal interactions is in the continuum limit described by an integro-differential equation, the so called neural field equation. Assuming that there are two stable states of the system, "active" and "inactive", it can be shown that there exists a traveling wave solution connecting the two states. Taking extrinsic random influences into account, we realise

the one-dimensional neural field equation as a function valued stochastic evolution equation and analyse the stability and the speed of the traveling wave.

### **Matti Leimbach (RTG TU Berlin)**

Title: Stochastic Stabilization

Abstract: It is well known that specific deterministic systems, which exhibit unbounded growth, can be stabilized by additive white noise in the sense that there exists an invariant probability measure. The existence of a Lyapunov function is almost sufficient to prove this trait. However, finding or constructing such a function is difficult and sometimes involves some guess-and-check. In order to gain a better insight of the mechanisms at work we investigate an example and give a Lyapunov function.

### **Marvin Müller (RTG TU Berlin)**

Title: An SPDE Model for the Limit Order Book

Abstract: Free boundary problems allow for modeling of multi-phase systems with separating hyperplanes evolving in time. We consider buy- and sell-side of the limit order book as such a two-phase system and introduce an infinite dimensional model based on a generalized stochastic Stefan problem. In contrast to their deterministic counterparts, stochastic free boundary problems are much less studied and understood. We apply a change of coordinates to rewrite the problem into a stochastic evolution equation and discuss existence, uniqueness and continuity of local solutions in a mild, weak and strong sense.

### **David Prömel (RTG HU Berlin)**

Title: Pathwise stochastic integration for model free finance

Abstract: In a recent series of papers, Vovk has introduced a model free, hedging based approach to mathematical finance that uses arbitrage considerations to examine which properties are satisfied by "typical price paths". One of the most important result is that typical price paths possess a nontrivial quadratic variation, justifying the use of Föllmer's pathwise Itô calculus in model free finance. Using only arbitrage arguments, we develop an integration theory: We show that it is possible to define a pathwise Itô type integral for typical price paths which are assumed to be càglàg with restricted jumps. This leads to a pathwise integral in the spirit of Föllmer, except that we allow multidimensional integrators. In order to give an axiomatic justification for the use of Lyons' rough path integrals in model free finance, we construct the required iterated integrals for typical continuous prices paths. Furthermore, we discuss a pathwise Tanaka formula and the existence of local times in this setting.

Joint work with Nicolas Perkowski.

## **Sebastian Riedel (postdoc TU Berlin)**

Title: A Jain-Monrad criterion for rough paths

Abstract: We present a novel criterion for the existence of Gaussian rough paths in the sense of Friz-Victoir. It is formulated in terms of a covariance measure structure together with a classical condition due to Jain-Monrad. It turns out that this condition is easy to check in many examples, ranging from (bi-)fractional Brownian motion to processes given as random Fourier series. We discuss several implications such as Cameron-Martin path regularity, the existence of a pathwise stochastic calculus, Itô-type exponential integrability for stochastic integrals and non-Markovian Hörmander theory.

This is joint work with Peter Friz (TU and WIAS Berlin), Benjamin Gess (HU Berlin) and Archil Gulisashvili (Ohio University)

## **Pierre-François Rodriguez (ETH Zurich)**

Title: On percolation of Gaussian free field level-sets in high dimension

Abstract: We consider level-set percolation for the Gaussian free field on  $\mathbb{Z}^d, d \geq 3$ , which is a percolation model with long-range dependence. It is presently known that the level-set above level  $h \in \mathbb{R}$  exhibits a non-trivial percolation phase transition, as  $h$  varies, for all dimensions  $d \geq 3$ . We compute the principal asymptotic behavior of the corresponding critical parameter  $h_*(d)$  as  $d$  goes to infinity.

Joint work with Alexander Drewitz.

## **Alexandros Saplaouras (TU Berlin)**

Title: Lévy processes under Sublinear Expectation Spaces

Abstract: After the introduction of G-Brownian Motion, Hu and Peng proceed to the definition of Lévy Processes under Sublinear Expectation Spaces (G-Lévy processes) [1]. In the current talk, this new framework will be described. Moreover, the differences comparing to the case of G-Brownian Motion will be briefly discussed and mainly the problems that have to be overcome in order to be able to define a stochastic integral with respect to a G-Lévy process.

Reference [1] Hu and Peng. G-Lévy Processes under Sublinear Expectations, ArXiv 2009  
<http://arxiv.org/abs/0911.3533>

## **Atul Shekhar (Berlin Mathematical School TU)**

Title: Lévy Khintchine formula for Rough Paths

Abstract: d-dimensional Lévy processes can be considered as rough paths with the iterated integrals understood in suitable geometric sense. The expected value of the signature, i.e. the ensemble of all iterated integrals, which generically describes its law, is shown to admit an

explicit expression. In fact the resulting formula is precisely of Levy-Kintchine type with its (d-dimensional) argument omitted.

### **Moritz Voss (TU-Berlin)**

Title: On optimal investment in a price impact model

Abstract: In this talk we present a price impact model which specifies the dynamics of bid and ask prices directly while retaining the possibility to specify market depth, tightness and resilience (the stylized facts observed in illiquid financial markets). We discuss the problem of optimal investment in this model providing a proof for the absence of arbitrage opportunities and the existence of optimal strategies. This work is part of my thesis project supervised by Peter Bank.

### **Maite Wilke Berenguer (TU Berlin)**

Title: On the stability of a dynamical system arising in telecommunication networks.

Abstract: We consider a queueing model that arises in the analysis of certain telecommunication networks. Three independent Poisson sources send their messages through two channels according to a Largest-Processing-Time-First scheduler. This simple model exhibits unusual behaviour as patterns appear that raise the critical stability threshold and induce the existence of meta-stable sets depending on the parameter of the Poisson sources. We will consider the instability of the model when this parameter exceeds a the named threshold.