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Adaptive Discretisation of Shell Problems

A seven parameter Reißner-Mindlin shell kinematic is employed for a elastoplastic material with hardening. The resulting nonlinear minimization problem is discretised within a finite element method on the mid surface of the shell. A posteriori error estimates are discussed and related adaptive algorithms are presented. Numerical examples illustrate the theoretical results.

1. Model Problem

The model problem investigated is a cylindrical shell with the thickness t. It occupies the domain $\mathcal{S}(z,\varphi,r) = \{((1+r)\cos\varphi, -(1+r)\sin\varphi, z)\}$, $-t/2 < r < t/2$, $0 < z < L$, $0 < \varphi \leq 2\pi$ and r being the distance from shell mid surface $\mathcal{S}(r=0)$. The displacement U^{3D} on $\mathcal{S}$ is given in terms of functions on the mid surface. The resulting displacement is used in elastoplasticity for small displacements. The displacement U^{3D} is described by a set of 7 functions, or a 7 parameter kinematics,

$$U^{3D} = (u - r\theta)\,\mathrm{e}_z + (v - r\phi)\,\mathrm{e}_\varphi + (w - r\psi - r^2\eta)\,\mathrm{e}_r. \tag{1}$$

2. Principle of Virtual Work

For an arbitrary material law, the principle of virtual work reads

$$\int_{\mathcal{S}} \sigma : \epsilon(V^{3D})\,d\mathcal{S} - \int_{\mathcal{S}} f \cdot V^{3D}\,d\mathcal{S} = 0 \quad (V^{3D} \in H^1(\mathcal{S})^3). \tag{2}$$

As the testfunctions obey the kinematics, this yields to a strong formulation consisting of seven partial differential equations,

$$\int_{-t/2}^{t/2} \frac{1}{\chi}(f_z + \sigma_{zz,z} + \chi\sigma_{z\varphi,\varphi})\,dr = 0, \qquad \int_{-t/2}^{t/2} \frac{1}{\chi}(rf_z - r\sigma_{zz,z} - r\chi\sigma_{z\varphi,\varphi} + \sigma_{zr})\,dr = 0,$$

$$\int_{-t/2}^{t/2} \frac{1}{\chi}(f_\varphi + \sigma_{\varphi z,z} + \chi\sigma_{\varphi\varphi,\varphi} + \chi\sigma_{\varphi r})\,dr = 0, \qquad \int_{-t/2}^{t/2} \frac{1}{\chi}(rf_\varphi - r\sigma_{\varphi z,z} - r\chi\sigma_{\varphi\varphi,\varphi} + (1-r\chi)\sigma_{\varphi r})\,dr = 0,$$

$$\int_{-t/2}^{t/2} \frac{1}{\chi}(f_r + \sigma_{rz,z} + \chi\sigma_{r\varphi,\varphi} - \chi\sigma_{\varphi\varphi})\,dr = 0, \qquad \int_{-t/2}^{t/2} \frac{1}{\chi}(rf_r - r\sigma_{rz,z} - r\chi\sigma_{r\varphi,\varphi} + r\chi\sigma_{\varphi\varphi} + \sigma_{rr})\,dr = 0,$$

$$\int_{-t/2}^{t/2} \frac{1}{\chi}(r^2 f_r - 2r\sigma_{rr} + r^2(\sigma_{rz,z} - \chi\sigma_{\varphi\varphi} + \chi\sigma_{r\varphi,\varphi}))\,dr = 0.$$

3. A Posteriori Error Indicator

A residual-based a posteriori error estimate can be obtained from the strong formulation above with element contributions η_T and edge contributions η_E. The constants in this reliable a posteriori error estimate depend on the thickness, and efficiency is still open. The local contributions are chosen as error indicator for the adaptive refinement algorithm. Both η_T and η_E are given for arbitrary discrete stresses Σ

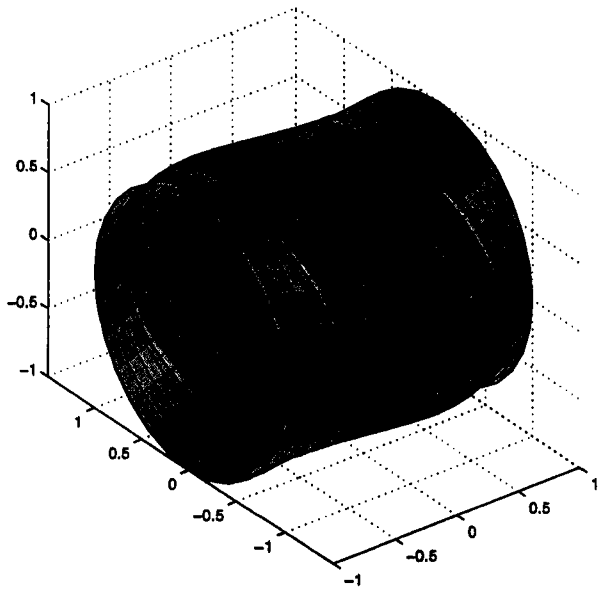
$$
\begin{aligned}
\eta_T^2 = h_T^2 \int_T \Bigg\{ & \left| \int_{-t/2}^{t/2} (f_1 - \Sigma_{zz,z} - \chi \Sigma_{\varphi z,\varphi})(1+r)dr \right|^2 \\
& + \cdots \\
& + \left| \int_{-t/2}^{t/2} (r(\Sigma_{zz,z} + \chi \Sigma_{\varphi z,\varphi}) - \Sigma_{rz})(1+r)dr \right|^2 \\
& + \cdots \\
& + \left| \int_{-t/2}^{t/2} (r^2(\Sigma_{rz,z} + \chi \Sigma_{r\varphi,\varphi} - \chi \Sigma_{\varphi\varphi}) - 2r\,\Sigma_{rr})(1+r)dr \right|^2
\end{aligned}
$$

$$
\begin{aligned}
\eta_E^2 = h_E \int_E \Bigg\{ & \left| \int_{-t/2}^{t/2} \left[\Sigma \cdot \begin{pmatrix} n_z \\ \chi n_\varphi \\ 0 \end{pmatrix} \right] (1+r)dr \right|^2 \\
& + \left| \int_{-t/2}^{t/2} \left[\Sigma \cdot \begin{pmatrix} n_z \\ \chi n_\varphi \\ 0 \end{pmatrix} \right] r\,(1+r)dr \right|^2 \\
& + \left| \int_{-t/2}^{t/2} \left[\Sigma_{rz} \cdot n_z + \chi \Sigma_{r\varphi} \cdot n_\varphi \right] r^2\,(1+r)dr \right|^2
\end{aligned}
$$

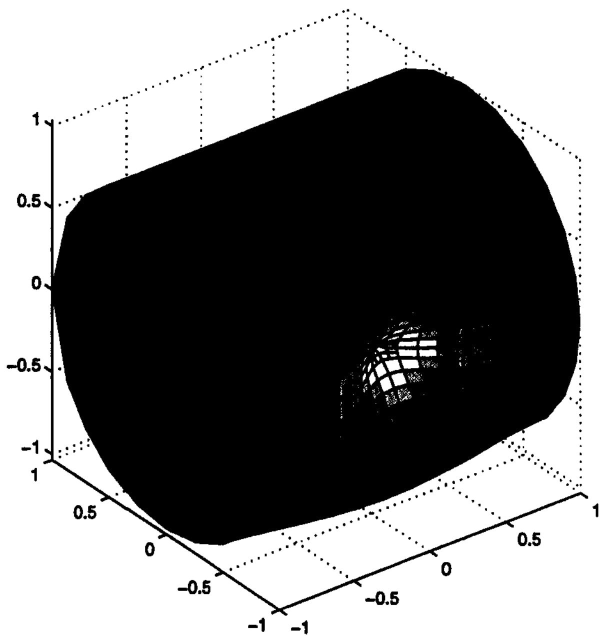
$$\chi = 1/(1+r)$$

4. Numerical Experiments

We discretised the model problem with Q1 finite elements on rectangular grids aligned both to the φ- and z-axis. Due to the adaptive refinement algorithm the discretisation allows hanging nodes. We have done numerical experiments with a point load, a radial load as well as a load tangential to the midsurface. Shown here are one example with point load as well as one example with radial load.



point load



radial load

The brightness indicates van Mises stresses $||\text{dev}\Sigma||$. The radial load is given by $f = (z^2 - 1)^4 cos(2\varphi)e_r$. It can be seen that for both loads the adaptive refinement algorithm results in finer meshes at areas with high changes in stresses.

Further work will be done in applying the a posteriori error indicator to more general material laws in elastoplasticity and a broader range of finite element discretisations.

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