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Problem 1. Show that in the discrete setting, inf-sup constants are singular values.

Problem 2. Compute the inf-sup constant of the global bilinear form in terms of α and β in the Brezzi splitting lemma.

Problem 3. Why is the primal dPG problem equivalent to the standard formulation of the Poisson model problem?

Problem 4. Show for all $q \in H(\operatorname{div}, \Omega)$ and $v \in H_0^1(\Omega)$ that

$$\|q - \nabla v\|^2_{L^2(\Omega)} + \|\operatorname{div} q\|^2_{L^2(\Omega)} \approx \|q\|^2_{H(\operatorname{div},\Omega)} + \|\nabla v\|^2_{L^2(\Omega)}.$$

Problem 5. Prove that

$$\inf_{0\neq(v,q)\in H_0^1(\Omega)\times H(\operatorname{div},\Omega)} \frac{\|q - \nabla v\|^2_{L^2(\Omega)} + \|\operatorname{div} q\|^2_{L^2(\Omega)}}{\|\nabla v\|^2_{L^2(\Omega)} + \|q\|^2_{H(\operatorname{div},\Omega)}} \lesssim \inf_{0\neq(v,q)\in H_0^1(\Omega)\times H(\operatorname{div},\Omega)} \frac{\|b(v, q\cdot\nu; \bullet)\|^2_{H^{-1}(\Omega)}}{\|\nabla v\|^2_{L^2(\Omega)} + \|q\|^2_{H(\operatorname{div},\Omega)}}.$$

Problem 6. The linear functional

$$\Lambda(q) := \int_E q \cdot \nu_E \,\mathrm{d}s$$

for an edge $E \subset \partial\Omega$ of a triangle Ω is well-defined for $q \in H^1(\Omega;\mathbb{R}^2)$. Prove that Λ has no continuous extension to $\hat{\Lambda} : H(\operatorname{div},\Omega) \to \mathbb{R}$.

Problem 7. Study the domain of the Fortin interpolation operator in the mixed FEM literature.