Solving $\mathbb{R}$-linear (P)DAEs by monomial basis extension

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Consider the following IVP on the unit interval I:

$$\ddot{u} + u = 0 \quad u(0) = u_0 \quad \dot{u}(0) = u_1.$$

The family $\left\{e_k : t \mapsto \frac{t^k}{k!}\right\}_{k\in\mathbb{N}}$ is a Schauder basis for $L^2(I)$;
and by expressing $u = \sum_{k\in\mathbb{N}} [u]_k e_k$ in this basis, one obtains the recurrence relation

$$[u]_0 = u_0 \quad [u]_1 = u_1 \quad [u]_{k+2} = -[u]_k.$$

Not only are these equations efficiently solved, they allow computation to be carried out purely symbolically, if desired. Generalizations to higer dimensions, orders and non-linear equations are discussed and a comparison with other methods is presented.