Finite Elements for Maxwell Equations

(A personal journey)

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This year marks the 150 years of Maxwell's Equations. James Clerk Maxwell presented his important finding to the British Royal Society in 1864, and published his work in the Philosophical Transactions of the Royal Society of London in 1865.

Instead of attempting (an impossible) overview of numerical methods for Maxwell equations, I will give a very personal story of my own struggle with the subject. My starting point for the Maxwell adventure was a very special class of Finite Element (FE) methods – the so-called hp-adaptive elements in which both element size h and polynomial order p are varied locally in a judicious way to resolve singularities and boundary layers occurring in the solution. The hp-adaptive methods deliver provided the underlying formulation guarantees stability. I will discuss two different ways to accomplish it. The first builds on a standard Galerkin formulation in which the stability comes from the use of FE spaces that form an exact sequence. The structure is intimately related to the linear dependence of Maxwell Finite Elements for Maxwell Equations which has to be reproduced at the discrete level. The resulting hp methodology for Maxwell equations is very competitive in a low frequency regime and has been applied to a number of difficult engineering problems including modeling of induction tools and scattering problems [1,2].

The second approach builds upon the Discontinuous Petrov-Galerkin (DPG) method being developed only very recently [3]. The stability comes from the use of optimal test functions computed on a fly. With the new DPG methodology, we are able to venture into more challenging and exotic versions of Maxwell equations including cloaking and metamaterials.

[1] L. Demkowicz, L., ``*Computing with hp Finite Elements. I.One- and Two-Dimensional Elliptic and Maxwell Problems*", Chapman & Hall/CRC Press, Taylor and Francis, October 2006.

[2] L. Demkowicz, J. Kurtz, D. Pardo, M. Paszynski, W. Rachowicz and A. Zdunek, ``*Computing with hp Finite Elements. II. Frontiers: Three-Dimensional Elliptic and Maxwell Problems with Applications*", Chapman & Hall/CRC Press, Taylor and Francis, August 2007.

[3] ``*An Overview of the DPG Method*", ICES Report 2013/2, also in ``*Recent Developments in Discontinuous Galerkin Finite Element Methods for Partial Differential Equations*", eds: X. Feng, O. Karakashian, Y. Xing, IMA Publications, Springer-Verlag, 2013.