# On a conjecture of Toponogov on complete convex surfaces

## *Klingenberg, Wilhelm*

In 1995, Victor Andreevich Toponogov authored the following conjecture: "Every smooth strictly convex and complete classical surface of the type of a plane has an umbilic point, possibly at infinity". He proceeded to prove this in case the gradients of mean and Gauss curvatures on the surface under consideration are bounded. In our talk, we will outline a proof, in collaboration with Brendan Guilfoyle, of the general case. Namely we prove that (a) the Fredholm index of an associated Riemann Hilbert boundary problem for holomorphic discs is negative. Thereby, (b) no such solutions may exist for a generic perturbation of the boundary condition (these form a Banach manifold under the assumption that the Conjecture is incorrect). Finally, however, (c) the geometrization by a neutral metric of the associated model allows for Mean Curvature Flow with mixed Dirichlet - Neumann boundary conditions to generate a holomorphic disc from an initial spacelike disc. This completes the indirect proof of said conjecture as (b) and (c) are in contradiction.