

Normal form for periodic FPU chains

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Abstract

We investigate normal forms of FPU chains with N particles and periodic boundary conditions, and we distinguish between odd and even N .

We prove that any chain with an odd number of particles admits a Birkhoff normal form up to order 4 near the equilibrium position, and we obtain an explicit formula of the Hessian of its Hamiltonian at the fixed point. Furthermore we show that Kolmogorov's nondegeneracy condition is satisfied for almost all parameter values, such that the classical KAM theorem can be applied near the equilibrium position.

For chains with an even number of particles, we construct a resonant normal form up to order 4, which we show to be integrable.