

# Topology II

## Exercise sheet 4

**Exercise 1.**

Finish the prove of Theorem 3.7 (i.e. verify the exactness of the long exact sequence) and deduce from it Corollary 3.8.

**Exercise 2.**

- (a) Reformulate Corollary 3.10 using reduced homology groups.
- (b) Compute again the homology groups of spheres by using Theorem 3.13.

**Bonus:** Is there a way to compute the homology groups of all closed surfaces using Theorem 3.13?

**Exercise 3.**

Let  $x_0 \in X$  be a point.

- (a)  $\tilde{H}_k(X) \cong H_k(X)$  for  $k \geq 1$ .
- (b)  $\tilde{H}_0(X) \cong \mathbb{Z}^{n-1}$ , where  $n$  is the number of path components of  $X$ .
- (c)  $\tilde{H}_k(X) \cong H_k(X, \{x_0\})$ .

**Exercise 4.**

Let  $X$  be a path-connected space. Show that  $H_0(X) \cong \mathbb{Z}$  and that  $H_1(X)$  is isomorphic to the abelization  $\pi_1^{ab}(X)$  of the fundamental group of  $X$ .

**Bonus exercise.**

Construct a pair of spaces  $(X, A)$  such that  $\tilde{H}_k(X/A)$  is **not** isomorphic to  $H_k(X, A)$ .

**Bonus exercise.**

Fill in the details of the proof of Theorem 3.5. from the lecture and draw a 2- and a 3-dimensional picture visualizing it.