

Topology II

Exercise sheet 15

Exercise 1.

Compute the intersection form

$$I: H_2(T^4) \times H_2(T^4) \longrightarrow \mathbb{Z}$$

of the 4-torus.

Exercise 2.

- (a) Use Corollary 8.3 and the intersection form to compute the ring structure of $\mathbb{C}P^n$.
- (b) Show that $\mathbb{C}P^{2m}$ admits no orientation reversing diffeomorphism.
- (c) Show that any map $\mathbb{C}P^{2m} \rightarrow \mathbb{C}P^{2m}$ has a fixed point.

Hint: Use the Lefschetz fixed point theorem.

Bonus: What can you say about maps $\mathbb{C}P^{2m+1} \rightarrow \mathbb{C}P^{2m+1}$?

Exercise 3.

Show that $\mathbb{C}P^2 \# (-\mathbb{C}P^2)$ is homeomorphic to an S^2 -bundle over S^2 and use this to deduce results about its homotopy groups.

Exercise 4.

Let M be a compact contractible n -manifold, then its boundary ∂M has the same homology and cohomology as the $(n-1)$ -sphere S^{n-1} .