WS 2023/24 Marc Kegel

Contact Geometry

Exercise sheet 11

Exercise 1.
Contact $(+1)$-surgery on a stabilized Legendrian knot K yields an overtwisted contact manifold.
Hint: Describe a Legendrian knot J in the complement of K and show that J, seen as a knot in $K(+1)$, violates the Bennequin bound. Can you explicitly describe an overtwisted disk in $K(+1)$?

Exercise 2.

(a) We say that a surgery is an **integer** surgery, if the surgery coefficient is an integer. Show that this notion is independent of the choice of the longitude.

(b) Any integer surgery corresponds to a 4-dimensional handle attachment.

(c) Every closed, connecnted, oriented 3-manifold bounds a compact orientable 4-manifold.
Hint: Use (b) together with the Lickorish–Wallace theorem.

(d) Describe connected sums in surgery diagrams.

(e) The lens spaces $L(p,q)$ as defined in Exercise 5 on Sheet 2 is diffeomorphic to the result of $-p\mu + q\lambda_S$ surgery on the unknot, where λ_S denotes the Seifert longitude of the unknot.
Hint: Show that the group action yielding the lens space preserves the splitting of S^3 into two solid tori and compute the new gluing maps.

(f) $\mathbb{R}P^3$ is diffeomorphic to $L(2,1)$.

(g) For every $n \in \mathbb{Z}$, $L(p,q)$ is diffeomorphic to $L(p, q+np)$.
Hint: Perform an n-fold twist along the Seifert disk of the unknot.

Exercise 3.
Prove Theorem 7.2 from the lecture.
Hint: Compute the surgery framing in the local model of the Weinstein 2-handle with respect to the contact framing.

Exercise 4.
Let K be a Legendrian unknot that is stabilized once positive and once negative. Show that contact (-1)-surgery on K yields a virtually overtwisted contact structure on a lens space.
Hint: Describe a Legendrian knot J in the complement of K such that J bounds an immersed overtwisted disk in $K(-1)$ that yields an embedded overtwisted disk in the universal covering.

Bonus exercise.
Let L be a Legendrian link in (S^3, ξ_{st}) along which we perform contact (± 1)-surgery to obtain a contact manifold (M, ξ). Let K be a Legendrian knot in the complement of L. Then K represents also a Legendrian knot in (M, ξ). Describe an algebraic criterion (depending only on the linking numbers of the Legendrian knots, the surgery coefficients and the classical invariants) that is equivalent to the statement that K is nullhomologous in (M, ξ). In that case, compute the Thurston–Bennequin invariant of K in (M, ξ).

This sheet will be discussed on Wednesday 24.1. and should be solved by then.