WS 2023/24 Marc Kegel

Contact Geometry

Exercise sheet 7

Exercise 1.

(a) The boundary of a standard neighborhood of a transverse knot is not convex.

(b) Every transverse knot admits a tubular neighborhood whose boundary is convex.

(c) Let S be a surface whose characteristic foliation admits a flow line from a negative singularity to a positive singularity. Can S be convex?

Exercise 2.
We consider $T^2 \times \mathbb{R}$ with 1-forms

$$\alpha_1 := \sin(\pi y)dx + 2\sin(\pi x)dy + (2\cos(\pi x) - \cos(\pi y))dz$$
$$\alpha_2 := \sin(\pi y)dx + (1 - \frac{1}{K}\cos(\pi x))\sin(\pi x)dy + (\cos(\pi x) - \frac{1}{K}\cos(2\pi x) - \cos(\pi y))dz.$$

Here x and y are coordinates on T^2 that are induced by $T^2 = \mathbb{R}^2/\mathbb{Z}^2$ and z is a coordinate on $\mathbb{R}$.

(a) Show that for a suitable choice of $K \in \mathbb{R}$ these 1-forms are contact forms inducing contact structures ξ_1 and ξ_2.

(b) Compute the characteristic foliations on $T^2 \times 0$ and draw them.

(c) Find the singularities of the foliations and compute their indices and divergences.

(d) Are these characteristic foliations homeomorphic? Are they diffeomorphic?

Exercise 3.
Let L be a Legendrian knot in $(\mathbb{R}^3, \xi_{st})$ and D one of its front project. We denote by F the Seifert surface of L that is obtained by applying the Seifert algorithm to the diagram D.

(a) Show that $\mathrm{tb}(L) \leq 2g(F) - 1$.

(b) Show that $\mathrm{tb}(L) \pm \mathrm{rot}(L) \leq 2g(F) - 1$.

(c) Can you deduce from (a) and (b) the Bennequin inequality for Legendrian knots in $(\mathbb{R}^3, \xi_{st})$? If yes give a prove. If no give a counterexample.

(d) Use (b) to give a contact geometric proof that the right-handed trefoil is not smoothly isotopic to the unknot.

(e) Use (b) (or directly the Bennequin inequality) to given an alternative proof of Exercise 4 (d) from Sheet 5, i.e. show that for every natural number $g \in \mathbb{N}_0$ there exist a knot K_g that has genus g.

This sheet will be discussed on Wednesday 13.12. and should be solved by then.