WS 2023/24 Marc Kegel

Contact Geometry

Exercise sheet 8

Exercise 1.
Consider $M = T^2 \times \mathbb{R}$ with circle-valued coordinates φ and θ on the T^2-factor, and with z denoting the $\mathbb{R}$ coordinate. Let ξ be the contact structure given as the kernel of $d\varphi + zd\theta$ on M. Let S in M be the 2-torus $T^2 \times 0$.

(a) Describe the characteristic foliation of S and deduce that S is not convex.

(b) Find an explicit perturbation S' of S, such that S' is convex. Describe its characteristic foliation and dividing set.

Exercise 2.

(a) Describe and depict the characteristic foliation of the *standard* overtwisted disk, i.e. a disk of radius π in $(\mathbb{R}^3, \xi_{ot})$.

(b) Let (M, ξ) be a contact manifold that contains a Legendrian unknot with $\mathrm{tb} = 0$. Show that (M, ξ) contains a standard overtwisted disk.
Hint: Use the elimination Lemma and the Poincaré–Hopf index theorem

Exercise 3.

(a) Let S be an embedded convex surface in a contact manifold. Let G be a properly embedded graph which is non-isolating. Then there exists an isotopy of S, rel boundary, to a surface S' such that G is contained in the characteristic foliation of S'.
Hint: Generalize the prove from the lecture of the Legendrian realization principle for knots.

(b) Use (a) to show that if $S \neq S^2$ nd Γ_S has no component that bounds a disk in S, then a vertically invariant neighborhood of S is tight.
Hint: Consider the universal cover of S and lift a Legendrian realization of the 1-skeleton and a potential overtwisted disk near S to the universal cover. Use that to realize the overtwisted disk as a convex surface from which you can construct an embedding of a neighborhood of the overtwisted disk in a tight contact manifold contradicting the assumption.

Exercise 4.
Prove Theorem 4.14 from the lecture.

This sheet will be discussed on Wednesday 20.12. and should be solved by then.