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Exercise 1.

The **dunce hat** is the topological space obtained from a triangle by identifying the sides as suggested in Figure 1.

Compute the fundamental group of the Klein bottle and of the dunce hat using the Seifert–van Kampen theorem.



Figure 1: The dunce hat.

## Exercise 2.

Let G be any finitely presented group. Construct a path-connected topological space X whose fundamental group is isomorphic to G.

**Bonus:** Is the same true for groups that are not finitely presented?

# Exercise 3.

Show that  $S^n$  and  $\mathbb{R}P^n$  are compact *n*-dimensional (smooth) manifolds without boundary.

#### Exercise 4.

Show that the coordinate cross  $\{xy = 0\}$  in  $\mathbb{R}^2$ , the dunce hat, and the cone over  $\mathbb{R}P^2$  are not manifolds.

#### Bonus Exercise.

Show that the comb with infinitely many teeth is contractible, but that not every point of it is a strong deformation retract.

## Puzzle Exercise.

Describe the free group  $F_n$  with n generators explicitly as a subgroup of the free group  $F_2$  with 2 generators.

These exercises will be discussed in the session on Thursday, June 5.

 $\mathrm{SS}~2025$