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Homework Problems 3

Analysis and Geometry on Manifolds WS 06/07

due 16.11.2006

The following 3 *problems are your homework assignment.*

Problem 1
(a) Let M, N be differentiable manifolds. Show that $M \times N$ is a differentiable manifold with the product topology.
(b) Show that $S^1 \times S^1 \cong T^2$ are diffeomorphic, where T^2 is the torus discussed in the class.
(c) Show that the tangent space $T_{(p,q)}(M \times N) \cong T_pM \times T_qN$ in a canonical way, where $p \in M$ and $q \in N$. Explain that this induces a diffeomorphism $T(M \times N) \cong TM \times TN$.

Problem 2
Let X, Y, Z be differentiable vector fields on a differentiable manifold M, f be a differentiable real valued function. Show the following identities:
(a) $[[X, Y,], Z] + [[Y, Z], X] + [[Z, X], Y] = 0$
(b) $[X, fY] = f[X, Y] + X(f)Y$

Problem 3
Show the following facts on topological spaces and continuous maps:
(a) If X is compact and $A \subset X$ a closed subset, then A is a compact set.
(b) If X is Hausdorff and $K \subset X$ compact, then K is closed.
(c) If X is compact, Y is Hausdorff and $f : X \to Y$ is bijective and continuous, then its inverse $f^{-1} : Y \to X$ is also continuous, i.e. f is a homeomorphism.

The following problems will be discussed in the tutorials.

Problem 4
Discuss counterexamples in Problem 3, i,.e. find a $T1$-space X such that (b) is not satisfied, and a non-compact space X for which (c) is not true.

Problem 5
Let X, Y be smooth vector fields on a differentiable manifold M for which flow Φ_s, Ψ_t, respectively exists for all times. Show that the following statments are equivalent:
(i) $[X, Y] = 0$
(ii) $[\Phi_s, \Psi_t] := \Phi_t \circ \Psi_s \circ (\Phi_t)^{-1} \circ (\Psi_s)^{-1} = id_M$ for all $s, t \in \mathbb{R}$.