Conference “Stability Phenomena in Geometry and Mathematical Physics“

09-12 October 2018
Humboldt-Universität zu Berlin, Institut für Mathematik,
Rudower Chaussee 25, 12489 Berlin, rm. 1.115

Organizers: Batu Güneysu, Olaf Müller (HU)

Titles and Abstracts:

Bernd Ammann (Universität Regensburg)

Parallel spinors on Riemannian and Lorentzian manifolds

Abstract:
The talk describes results in joint articles with Klaus Kröncke, Olaf Müller, Hartmut Weiß, and Frederik Witt.

We say that a Riemannian metric on M is *structured* if its pullback to the universal cover admits a parallel spinor. All such metrics are Ricci-flat. The holonomy of these metrics is special as these manifolds carry some additional structure, e.g. a Calabi-Yau structure or a G_2 -structure. All known compact Ricci-flat manifolds are structured.

The set of structured Ricci-flat metrics on compact manifolds is now well-understood, and we will explain this in the first part of the talk.

The set of structured Ricci-flat metrics is an open and closed subset in the space of all Ricci-flat metrics. The holonomy group is constant along connected components. The dimension of the space of parallel spinors as well. The structured Ricci-flat metrics form a smooth Banach submanifold in the space of all metrics. Furthermore the associated premoduli space is a finite-dimensional smooth manifold, and the parallel spinors form a natural bundle with metric and connection over this premoduli space.

Lorentzian manifolds with a parallel spinor are not necessarily Ricci-flat, however the rank of the Ricci tensor is at most 1, the image of the Ricci-endomorphism is lightlike. Helga Baum, Thomas Leistner and Andree Lischewski showed the well-posedness for an associated Cauchy problem. Here well-posedness means that a (local) solutions exist if and only if the initial conditions satisfy some constraint equations.

We are now able to prove a conjecture by Leistner and Lischewski which states that solutions of the constraint equations on an n-dimensional Cauchy hypersurface can be obtained from curves in the moduli space of structured Ricci-flat metrics on an $(n-1)$-dimensional closed manifold.

Volker Bach (TU Braunschweig)

Stability of Matter in Absence or in Presence of the Radiation Field

Abstract:
In this talk the notion of stability of quantum mechanical of quantum field theoretical theories is reviewed. In this context, three levels of stability are discussed:
(1) semiboundedness, (2) semiboundedness proportial to the particle number, and (3) existence of a ground state at the bottom of the spectrum.

Christian Bär (Universität Potsdam)

Gromov's weak flexibility lemma and a counter-intuitive approximation result

Abstract:
In his book on partial differential relations Gromov formulates his weak flexibility lemma as an exercise. An explanation of this lemma and its proof will be given. As the main application we show that every Riemannian metric on a manifold can be C^1-approximated by a C^1-metric with curvature at least 1 almost everywhere in the sense of Alexandrov spaces. This is totally false for C^2-metrics. This is joint work with Bernhard Hanke.

Lydia Bieri (University of Michigan)

Stability of Aymptotically-Flat Systems in Mathematical GR

Abstract:
In Mathematical General Relativity (GR) the Einstein equations describe the laws of the universe. This system of hyperbolic nonlinear pde has served as a playground for all kinds of new problems and methods in pde analysis and geometry. Asymptotically-flat systems in GR are solutions of the Einstein equations tending to Minkowski spacetime at infinity. They model stars, clusters of stars, galaxies and related situations in physics. Many solutions to the Cauchy problem for the Einstein equations give interesting insights into gravitational waves (observed for the first time by LIGO in 2015 and several times since then). The full understanding of a large class of such solutions came with the breakthrough result by D. Christodoulou and S. Klainerman proving the global nonlinear stability of Minkowski spacetime. The borderline case for nonlinear stability in terms of decay of the data was obtained in my proof revealing different geometric-analytic structures and implications for physics. These proofs unravel important information on the structures of these spacetimes including gravitational radiation. In this talk, I will briefly review main ideas of the stability proofs and then explore new results concerning radiation.

Gilles Carron (Université de Nantes)

Volume growth estimates on complete Riemannian manifolds

Abstract:
Polynomial bound for the volume growth of geodesic ball of complete Riemannian manifolds is a first coarse estimate that could lead to a better understanding of the geometry at infinity. In this talk, I will try to survey different technics : comparison geometry, spectral theory, functional analysis, harmonic analysis.

Piotr T. Chruściel (Universität Wien)

Stationary Black holes with negative cosmological constant

Abstract:
I will present a construction of large families of stationary black holes, solutions of Einstein equations with or without matter sources and with a negative cosmological constant.

Zoubir Dahmani (University of Mostaganem, Algeria)

On a Class of Differential Systems of Arbitrary Order and Ulam Stabilities

Abstract:
This work deals with a class of systems of fractional differential equations. We begin by presenting new results for the existence and uniqueness of solutions for the problem. Then, we establish some sufficient conditions for the existence of at least of one solution for the systems. The Ulam stabilities are also present in this work. At the end, some illustrative examples are discussed.

Alberto Enciso (ICMAT, Madrid)

Knots and links in the equations of mathematical physics

Abstract:
We will survey several results about the emergence of knots and links in solutions to the 3D Euler and Schrödinger equations. In the Euler equation, knots arise in the study of the vortex lines (or vortex tubes) of a fluid, and their study in stationary problems has been mostly driven by old conjectures of Lord Kelvin and Vladimir Arnold. Connections with the problem of vortex reconnection for the 3D Navier-Stokes equations will be discussed too. In quantum mechanics, the first results concerning knotted nodal sets for eigenfunctions of Schrödinger operators on Euclidean 3-space (in fact, the Coulomb Hamiltonian) are due to Berry. We will also discuss a reconnection problem for the 3D Gross-Pitaevski equation. The talk will be based on joint work with M.A. García-Ferrero, D. Hartley, R. Lucà and D.l Peralta-Salas.

Felix Finster (Universität Regensburg)

Linear stability of rotating black holes

Abstract:
After a general introduction to black holes and wave equations in black hole geometries, I will report on recent results on the linear stability of Kerr black holes under perturbations of general spin. This is joint work with Joel Smoller.

Nadine Große (Albert-Ludwigs-Universität Freiburg)

Title: Boundary value problems on noncompact manifolds

Abstract:
We consider Poisson problems on manifolds with boundary and bounded geometry and assume that they have finite width (that is, that the distance from any point to the boundary is bounded uniformly). We include Robin boundary conditions. As an application, we establish the connection to the Poisson problem on certain domains in the plane and higher dimensional stratified spaces. In particular we get the well-posedness of strongly elliptic equations on domains with oscillating conical singularities, a class of domains that generalizes the class of bounded domains with conical points. This is joint work with Bernd Ammann (Regensburg) and Victor Nistor (Metz).

Jeff Jauregui (Union College, Schenectady, NY)

Semicontinuity of total mass and low-regularity convergence in general relativity

Abstract:
In general relativity, the ADM mass captures the total mass of an asymptotically flat spacelike hypersurface in a spacetime. Motivated by the conjectured near-rigidity of the positive mass theorem and by Bartnik's minimal mass extension conjecture, it is natural to study the interaction between the ADM mass and low-regularity convergence. We will discuss joint work with Dan Lee on establishing the lower semicontinuity of ADM mass for pointed C^0 convergence and Sormani--Wenger intrinsic flat convergence, which has shown promise for low-regularity problems in general relativity. We will make use of Huisken's isoperimetric mass concept.

Klaus Kröncke (Universität Hamburg)

Stability of Einstein manifolds under Ricci flow

Abstract:
An Einstein metric is called dynamically stable if any (suitably normalized) Ricci flow starting close to it exists for all time and converges to an Einstein metric close to the given one. In this talk, we review some recent stability results in the compact as well as in the noncompact case. Moreover, we discuss how this problem is linked to questions of positive scalar curvature rigidity, nonlinear stability in general relativity and singularity theory of Ricci flow.

Miguel Sánchez Caja (Universidad de Granada)

Splitting and stability of globally hyperbolic spacetimes with timelike boundary

Abstract:
Globally hyperbolic spacetimes with timelike boundary constitute the natural class of spacetimes where regular boundary conditions can be posed on its set of naked singularities. Motivated by this PDE viewpoint, the known smooth Cauchy orthogonal splitting for the case without boundary will be extended to the case with boundary. Thus, such a spacetime $\bar{M}$ with boundary ∂M becomes isometric to $\mathbb{R}$ x $\overline{\sum}$ (where $\overline{\sum}$ is a smooth spacelike Cauchy hypersurface with boundary) endowed with a metric such that the factors $\mathbb{R}$ and $\overline{\sum}$ become orthogonal; in particular, ∂M intersects orthogonally a foliation of $\bar{M}$ by Cauchy hypersurfaces.
In order to achieve this result, the stability of both, global hyperbolicity and Cauchy temporal functions, are proven first. Such properties allow one to circumvent the details of the boundary and have interest in its own right.

Based in joint work with L. Aké and J.L. Flores, arxiv:1808.04412

Robert Seiringer (IST Austria)

Quantum many-body systems with point interactions

Abstract:
We investigate the stability of quantum many-body systems with point interactions. In particular, we present a proof that a system of N fermions interacting with an additional particle via point interactions is stable if the ratio of the mass of the additional particle to the one of the fermions is larger than some critical value. For this impurity problem, we also show that the ground state energy of the system at given non-zero mean density differs from the one of the ideal gas by a term depending only on the density and the scattering length of the interactions, independently of N. While the general problem with more than one impurity remains open, we can show stability for the simplest such system, the one consisting of two fermions interacting with two (fermionic) impurities. (Joint work with T. Moser)

Anton Thalmaier (Université du Luxembourg)

Subelliptic Brownian motion and curvature in sub-Riemannian geometry

Abstract:
For sub-Riemannian manifolds, to give intrinsic meaning to geometric notions depending only on the sub-Riemannian structure, is a problem which attracted a lot of attention. We describe some recent work which is centered around the concept of curvature in sub-Riemannian geometry and relies on the study of subelliptic diffusion processes on sub-Riemannian manifolds.

Wilderich Tuschmann (KIT, Karlsruhe)

Moduli Spaces of Riemannian Metrics with Nonnegative Curvatures

Abstract:
I will report on general results and questions about spaces and moduli spaces of Riemannian metrics with non-negative Ricci or non-negative sectional curvature on closed and open manifolds, and present recent joint work with Michael Wiemeler. In particular, we construct the first classes of manifolds for which these spaces have non-trivial rational homotopy, homology and cohomology groups.

Giona Veronelli (Sorbonne, Paris)

Extent of balls, scalar curvature and metric spaces

Abstract:
We are interested in the problem of defining the scalar curvature of a non-smooth metric space, with a special attention to the case of Alexandrov spaces with lower bounded curvature.
On the one hand, we will present a characterization of the scalar curvature of a smooth n-dimensional Riemannian manifold based on the asymptotic expansion of the (n+1)-extent of its geodesic balls, the (n+1)-extent being the maximal total distance between n+1 points in the ball.
On the other hand, we will discuss to what extent this alternative definition of scalar curvature makes sense on metric spaces and we will compare it with other possible approaches.