Übungen zur Lorentzgeometrie und Mathematischen Relativitätstheorie

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Übungsblatt 2

Exercise 1: Foundations of causality theory

Let (M, g) be a spacetime. Show the push-up property for any three points of M, and show $J^+(p) \subset \overline{I^+(p)}$ for all $p \in M$.

Exercise 2: Domains of dependence

Let (M, g) be a spacetime and let $A \subset M$ be acausal. Show that $D(A)$ is causally convex and does not contain closed causal curves.

Exercise 3: Causal? Diamond-compact?

Let $a \in \mathbb{R}^+$, $u \in C^\infty(\mathbb{R}^2)$ and $f \in C^\infty((-1;1))$. Which are the conditions on a, u, f such that the following spacetimes are causal and/or diamond-compact? In the third example, try to answer the question for the concrete examples $f_1(x) := (1-x^2)^2$ und $f_2 := (1-x^2)^{1/4}$.

1. $\{x = (x_0, x_1) \in \mathbb{R}^{1,1} : |x_1| < 1 \wedge x \in I^+((-a, 0)) \cap I^-((a, 0))\}$;
2. $(\mathbb{R}^2, g_{1,1} + e^u dx_1^2)$;
3. $(\mathbb{R} \times (-1;1), -f^2(x_1)dx_0^2 + dx_1^2)$;
4. $\{(x_0, x_1) \in \mathbb{R}^{1,1} | x_0 \in (-a; a) \wedge x_0 + x_1 \in (-a; a)\}$.