Übungen zur Lorentzgeometrie und Mathematischen Relativitätstheorie

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Übungsblatt 3

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Exercise 1: Causality theory

Let (M, g) be a spacetime. Show:

1. $J(p, q)$ is causally convex for every two $p, q \in M$.
2. If (M, g) admits a Cauchy surface S, it is intersected by every generalized future curve.

Exercise 2: Cauchy time functions

Let t be a Cauchy time function on a spacetime (M, g). Then t is continuous. **Hint:** Show first $t \circ c$ continuous for every future causal curve c.

Exercise 3: Examples

Show that the following subsets D_n, A_n of semi-Euclidean spaces are n-dimensional spacetimes and moreover homogeneous and isotropic, i.e., show that the group of isometries of D_n (resp. A_n) acts transitively on $T^a D_n$ (resp. $T^a A_n$), where, for a Lorentzian manifold (M, g) and $a \in \mathbb{R}$, $T^a M := \{v \in TM | g(v, v) = a\}$:

1. $D_n := \{x \in \mathbb{R}^{1,n} | \langle x, x \rangle = 1\}$,
2. $A_n := \{x \in \mathbb{R}^{2,n-1} | \langle x, x \rangle = -1\}$,

Finally, check causality and diamond-compactness for D_n and A_n.