Übungen zur Lorentzgeometrie und Mathematischen Relativitätstheorie

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Übungsblatt 9

Exercise1: Adapted is proper

Let a semi-Riemannian (M, g) manifold and two spacelike submanifolds S_1, S_2 of M be given. Let $c : [0; b] \to M$ be a causal geodesic with $c(0) \in S_1$, $c(b) \in S_2$ and $c'(0) \in TS^\perp, c'(b) \in TS_2^\perp$. Show that the variational vector field of an (S_1, S_2)-proper variation of c is an adapted Jacobi vector field and that for every adapted Jacobi vector field X there is an (S_1, S_2)-proper variation of c with variational vector field X.

Exercise 2: More on Jacobi vector fields

Show that for any two nontangential Jacobi vector fields J, K along a geodesic , $g(J, K') - g(J', K)$ is constant. Prove also Lemma 3.27 of the lecture.

Exercise 3: A formula for Ricci curvature

Prove that in an n-dimensional Lorentzian manifold (M, g), for a null vector $v \in T_pM$ and $e_1, ..., e_{n-2}$ spacelike unit vectors that are orthogonal to each other and to v, we get $\sum_{i=1}^{n-2} g(R(v, e_i)e_i, v) = ric(v, v)$.

Exercise 4: Spherical symmetry and conformal flatness

Show that if a Riemannian manifold (M, g) is spherically sdymmetric, it is conformally flat, and even more, there is a conformal factor $u : M \to \mathbb{R}$ such that $(M, e^{2u}$ is isometric to an open set in some $\mathbb{R}^n$ (recall that there are flat manifolds *without* any open isometric embedding into some $\mathbb{R}^n$).

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