Systolic inequalities and Gromov's filling area conjecture

Olaf Müller*

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We will seek our path through Gromov's voluminous and book-ish article [1], which explores estimates connecting the volume vol(M) of a given Riemannian manifold M and its *systole* $\mathrm{sys}_1(M)$, which is defined by

$$\mathrm{sys}_1(M) := \inf\{l(c) | c \in C^1(\mathbb{S}^1, M) \text{ noncontractible}\},$$

where $l(c)$ is the length of the closed curve c. The main such estimate is

$$\mathrm{sys}_1(M) \leq 6(n+1)n^n\sqrt{(n+1)!}(\mathrm{vol(M)})^{1/\mathrm{n}}.$$

It turns out that these questions are intimately connected to filling problems of the following sort:
Given a circle $S := \mathbb{S}^1(r)$ of radius r, what is the infimum of volumes of surfaces Σ with $\partial\Sigma = S$ that satisfy the 'no-shortcut condition': For any two points p, q in S, at least one shortest curve between p and q in Σ remains in S. Gromov's conjecture is that the hemisphere provides the minimal solution to the problem, but this has been proven only if one restricts the topology of Σ to the disc, and it remains open in its generality up to now. Because of such interesting spin-offs and the breathtaking beauty of the proofs in Gromov's article, often connecting different branches of mathematics in a surprising way, this book is our main source, completed by a couple of more recent references in the second half of the seminar.
I consider previous knowledge of Topology 1 and Differential Geometry 1 necessary and recommend knowledge of Differential Geometry 2 — but attending this course parallelly to the seminar will certainly do the job and will be made possible by choosing the seminar time accordingly.

References

[1] Mikhail Gromov: *Filling Riemannian Manifolds*, J. Diff. Geom. 18 (1): 1 — 147 (1983)

*Institut für Mathematik, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin (Germany), Email: `mullerol@math.hu-berlin.de`