Prof. Dr. N. Perkowski
http://www.math.hu-berlin.de/~perkowsk
Institute of Mathematics
Stochastics

Functional Analysis: Main content

- **Basics of topology:** metric spaces, completeness (Baire's theorem), compactness (in general/metric spaces, total boundedness)
- **Normed spaces:** convergence of absolutely convergent series, compactness of unit ball, equivalence of norms in finite dimensions
- **Hilbert spaces:** Cauchy-Schwarz, polarization, (complete) orthonormal systems, Pythagoras' equality, Bessel's inequality, special case of separable Hilbert spaces, orthogonal projection
- **Spaces of continuous functions:** compactness (Arzela-Ascoli), separability (Stone-Weierstrass)
- **Lebesgue spaces:** Hölder, Minkowski, Young (convolution) inequalities, completeness, separability (separable measure spaces), dense subsets (step functions, C_c, C^\infty_c), convolutions, approximate identities, mollifiers
- **Bounded linear operators:** continuous iff bounded, operator norm, uniform boundedness principle/Banach-Steinhaus, inverse mapping theorem, closed graph theorem, compact operators, self-adjoint, spectral theorem
- **Duality:** topological dual, Hahn-Banach, linear bounded functionals separate points, reflexivity, identification of dual spaces (Hilbert, L^p, C_b(X,R)), weak(-*) topology, Banach-Alaoglu, sequential version of Banach-Alaoglu in separable spaces, optimization problems in Hilbert spaces
- **Fourier series:** periodic functions, Fourier inversion in L^2, Parseval, link to derivatives
- **Fourier transform:** Schwartz space, Riemann-Lebesgue, Fourier inversion on S, link to derivatives/convolutions, Plancherel, extension to L^2
- **Distributions:** basic idea, linear operations on distributions