What is this, and how can you help?

The following pages contain a short and dense article about a board game. I am writing this (and similar texts for some other games) for basically two reasons:

1. I want to popularise cool board games which are less popular than they should be.
2. There are many accounts (books, online pages etc.) that just give the rules. In order to encourage more people to give them a shot, I'd like to go into a little depth: elementary tactics, problems etc. Hopefully, this helps drawing some future players!

I'm a moderately advanced Go player (1 dan) but not nearly an expert on any of the games I am writing about. Therefore, I will be happy and very grateful for all kinds of feedback. If you think I am way off the mark, please tell me! Remember, the more specific your feedback, the more I can improve the article.

Here are some features that the text is still lacking, but ideally would have:

1. **Problems:** Please have a good look at the problems in the text. Are they well-posed? Do you have ideas for other and/or better problems? (Customarily, problems have unique solutions. I'm not even sure if my current problems have this property.)
2. **More heuristics:** good strategy games have heuristics that allow players to break up the complexity into more manageable pieces. There's not much literature on these games, so I've been starting out in the most simple fashion. If you are using other concepts in your games, please tell me!
3. **Example positions:** if you have encountered a particularly surprising move (by yourself, an opponent, or someone else), feel free to send me the position; most easily as screenshot or LittleGolem link.

I already got some feedback through LG and BGG, and the articles have greatly benefited from that. If you would like to comment, these are the best options:

- right in this thread,
- an email to `dploog@math.fu-berlin.de`. Please mention the game in the subject.

Many thanks for reading this! May 16, 2019

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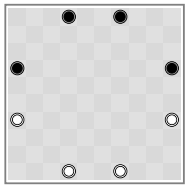

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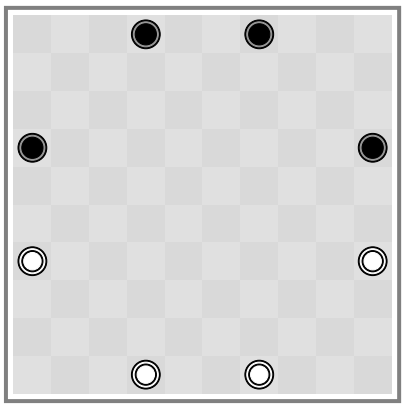
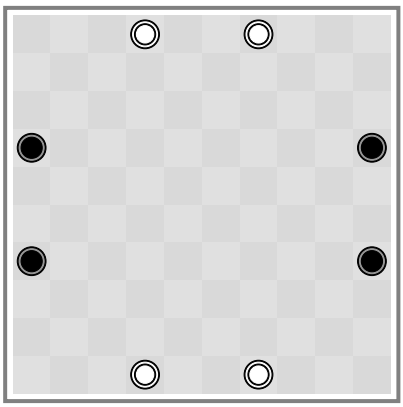
AMAZONS

The game takes place on a 10 × 10 board. Each player has four pieces, called *queens* or *amazons*. Additionally, many neutral pieces are needed, called (poison) *arrows*. They are denoted ■ in diagrams and can be of any other type or colour; GO stones or poker chips are well suited.



initial position



alternative cross position

White has the first turn. Recall that CHESS queens move in any of the eight straight directions, over any number of vacant squares. A turn consists of two actions, performed in this order:

1. **move**: make a CHESS queen move with an amazon of the player's colour;
2. **shoot**: make a CHESS queen move from that amazon's destination and place an arrow.

A player **loses** if unable to carry out a turn.

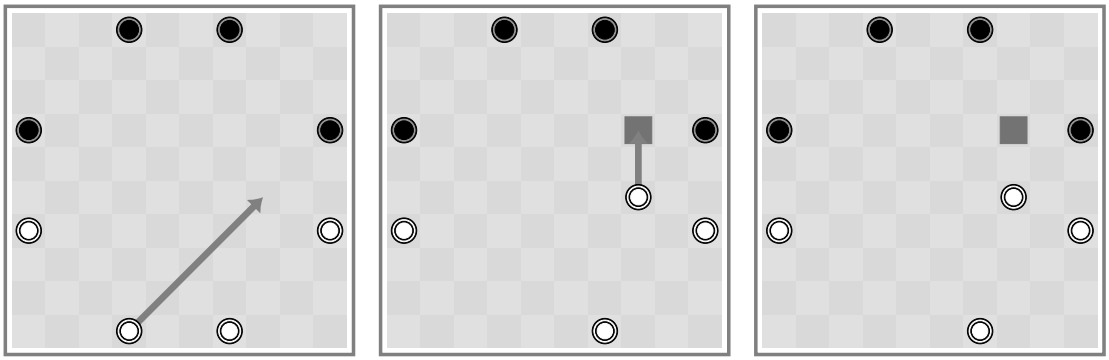
About the game

This game was introduced by Walter Zamkauskas from Argentina in 1988 with the title *Juego de las Amazonas* (The Game of Amazons). Today called AMAZONS, it is a modern classic. Even though AMAZONS is an abstract board game, its strong theme may have contributed to its popularity: this is a contest between two tribes of bow fighters competing for clan territory.

Walter Zamkauskas (1988)

Diagrams explaining the rules

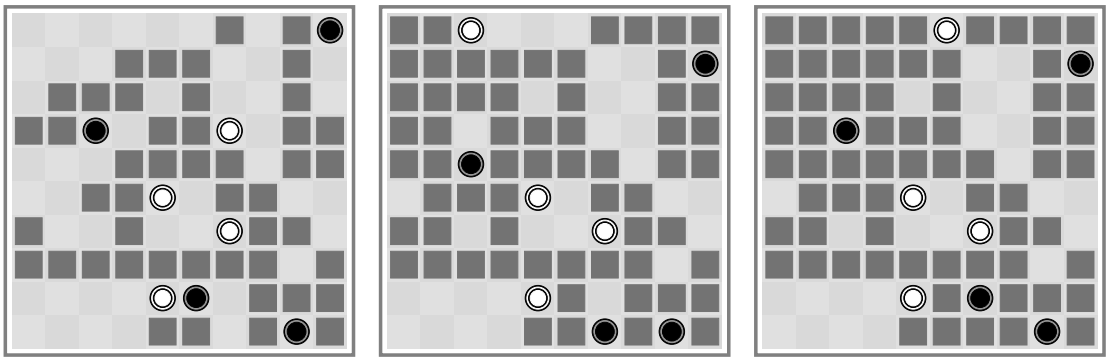
The following three diagrams show a possible first turn by White:



An amazon is moved like a CHESS *queen, and then an arrow is shot from its destination, also like a* CHESS *queen.*

In each turn, one arrow is added to the board, creating an obstacle for any amazon. Thus, space available for movement is shrinking throughout a match, and inevitably one player will be unable to move and shoot at some point, thereby losing the game.

However, in practice games aren't played until stalemate. As soon as the board is divided into regions where amazons of different colours are separated, the remaining turns can be counted. In other words, what matters are the territories belonging to each side. For beginners, it is worthwhile to play their first games until the very end, i.e. one player being unable to move.



A finished game: separated regions; *no more possible interaction;* *Black unable to move.*

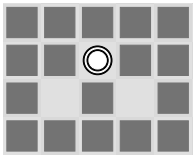
A typical AMAZONS game can be roughly structured in three phases:

1. Early game: *mobility*. Range and position of pieces; tentative regions are sketched out.
2. Midgame: *local dominance*. Areas take shape; mobility is still crucial.
3. Endgame.

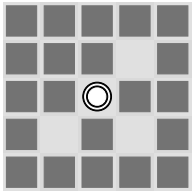
As happens with many board games, the final stage is most accessible: by then, the board will be divided into several separate areas of activity. This is why we begin our analysis of the game with a closer look at the end game. After that, we consider how to build territories, and what to do against them. Finally, we speak about mobility, a concept which is relevant throughout a game but especially in the opening.

Endgame

During play, it is reasonable to estimate territories by simply counting empty squares. However, sometimes not all those squares can be reached. The adjacent diagram shows the smallest position where this happens: the white queen has two empty squares available, but nonetheless she is reduced to one final turn.

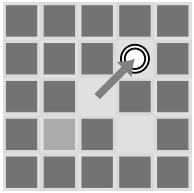


Only one turn.

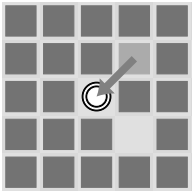


Three turns.

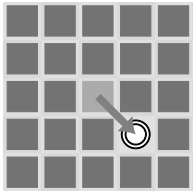
By contrast, this territory really is worth three points:



Turn 1

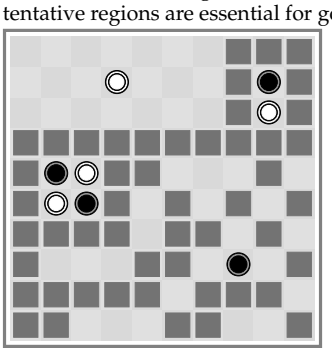


Turn 2

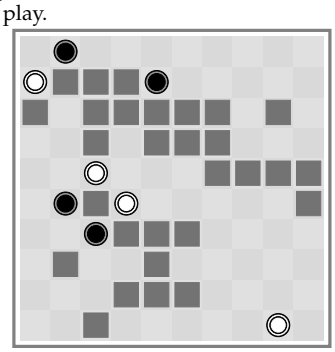


Turn 3

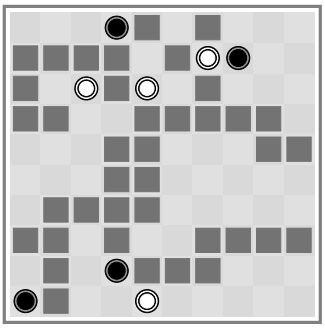
Three endgame problems:
It is a fun challenge to figure out how many points an odd-shaped territory really has. In actual games, this type of combinatorial endgame is not very prominent. However, careful counting of actual territories, and estimates of tentative regions are essential for good play.



Problem 1. Black to play.

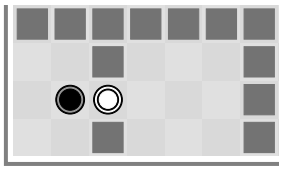


Problem 2. Black to play.



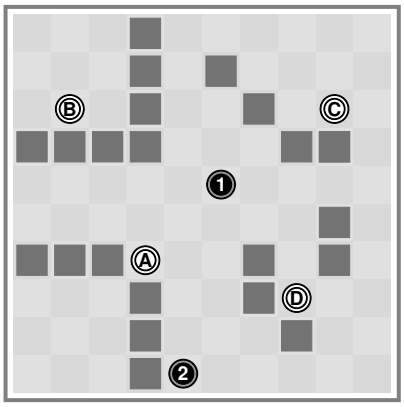
Problem 3. Black to play.

The door-guard. In the position to the right, the white amazon is guarding a territory of nine points. She could invade the black region but that would lose more white points than it costs Black. Therefore, the black queen secures five points even though her area is not tightly walled off. We call the white queen a *door-guard*, following the lead of Paul Yearout [A].



Territory

Potential and secure territory. The following artificial position contains four corner regions claimed by White but of very different quality:

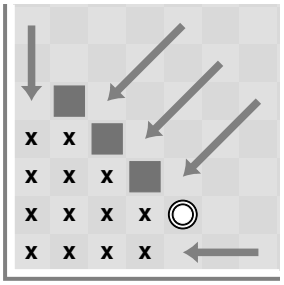


The two corners on the left side are secure white territory. There is a crucial difference between them: the queen Ⓐ is *active*; she can still participate in the game whereas Ⓑ is confined to her area.

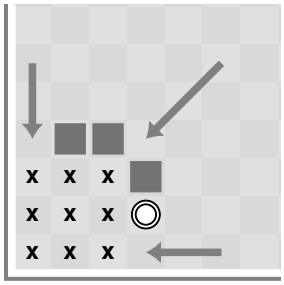
The top right corner is only potentially White's territory: ❶ can invade and Ⓒ can defend, so it depends on who is going to move first in this sector.

White cannot well turn the bottom right corner into secure territory in just one turn. This corner has weaker potential for White than the top right.

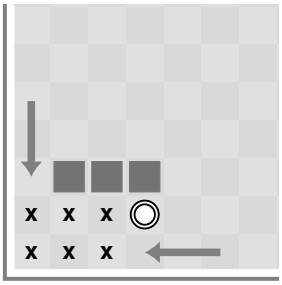
Building territory. The most efficient way to build territory is in the corners, for fewer arrows are needed to wall off a (more or less safe) region near the nooks. The following diagrams indicate schematically how a queen and three arrows outline a corner territory. Diagonal arrow shapes are very efficient but leave more lines of attack.



Corner size about 15, with five lines of attack.



Corner size about 9, with three lines of attack.



Corner size about 6, with two lines of attack.

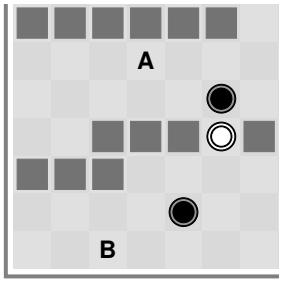
In the opening, it is rarely advisable to sketch smaller territories such as the middle and right-hand positions above: recall that the game begins with 92 empty squares and in each turn, one more square will be blocked off. Assuming an even and extremely close game taking 88 turns, each amazon would have moved $88/8 = 11$ times on average.

However, note that consigning an amazon to a very early territory of 12 or so points might still not be worth it: the womanpower shortage can easily lead to losses much larger than the initial gain. On the other hand, one should try to set up prospective, large territories.

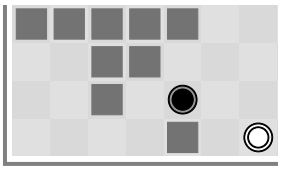
Dealing with opposing territory. There are three basic ways for how to tackle more or less secure territory of the opponent. Often, these are just threats to be taken into account — a typical move could consist of a positional threat (to isolate an opposing amazon) together with a territorial threat (to invade).

1. *Invade:* get an amazon into the opponent's area, wreaking havoc.
2. *Close off:* shut off the region, thus denying it to either side.
3. *Reduce:* shoot an arrow inside the area, reducing it by one point.

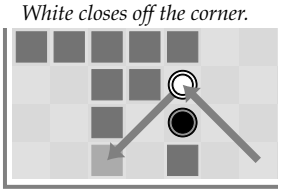
Note that the shooting an arrow inside an opposing territory is a very weak use of half a turn! If possible, this action should be spent elsewhere, at a location with positional effects. If forced to shoot a reduction arrow, try to find a location that will impede the opponent's movement in that area.

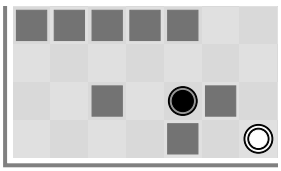


White can invade at either A *or* B*, and Black can only prevent one of these.*

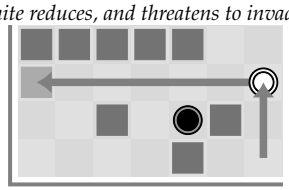


White closes off the corner.

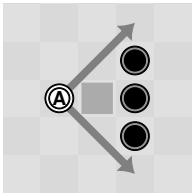




White reduces, and threatens to invade.

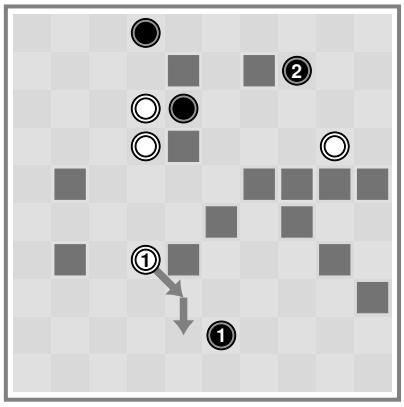


A useful invasion pattern. The threat to invade a region claimed by the opponent is important. If a player is about to execute such a threat, it will be crucial to make most out of the invading turn. The following pattern, which we call *the spearhead*, can help with this:



Assume that Black has a tentative territory around the pieces ●. If White moves a queen to Ⓐ and shoots its arrow right in front of it, as indicated, then there is no single move with which can Black deny both diagonal incursions. The arrow ■ is White's spearhead.

Here is an example from an actual game:

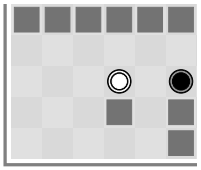


❶ threatens a large black bottom right corner. White cannot simply counter with an equally large bottom left corner because ❷ looms along the diagonal.

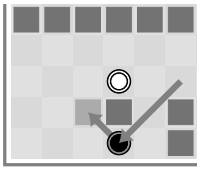
Therefore, White will attack the bottom right with ①. A good move is *d4-e3/e2*, enabling a full invasion while increasing the potential of the bottom left area. Here, White is using ❶ as spearhead.

Black would probably lose by defending the bottom right, and instead has to play more aggressively.

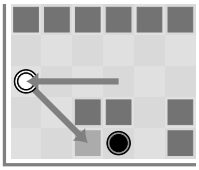
Counter-threats. *Do not stubbornly cling to your territories!*
This is a principle also familiar from many other games: if you consistently react obediently to threats, then you will lose. Instead always ponder if a frightful looking move really is forcing, and how you can resist. Here is a small scale example:



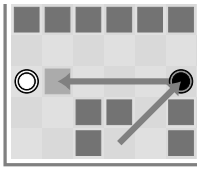
A corner position.



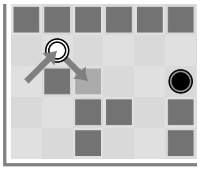
Black attacks.



White defends.



Black attacks.



White defends.

Assume that White insists on defending the corner territory. Dutifully replying to Black's threats, the area has shrunk to a measly six points. To make matters worse, Black is now able to establish an area of similar size with just one more move!

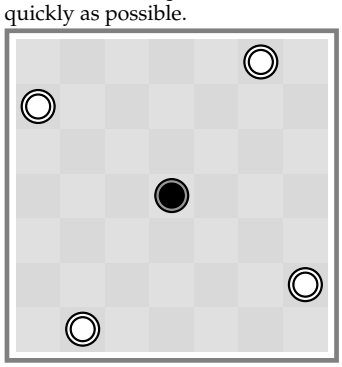
For comparison: if White would have played first in this position, the territory would have been at least twice the size. In other words, the reaction to Black's initial attack should be a White move somewhere else on the board, worth at least 7 points (either offensively or defensively).

Versatility is a virtue in AMAZONS. It often happens that threats and counter-threats lead to exciting territorial swaps!

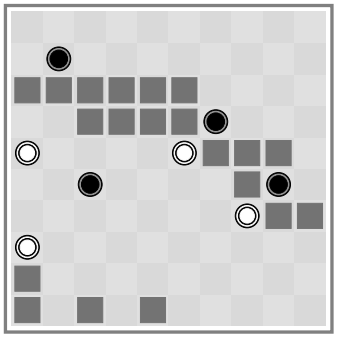
Material advantage vs. territory size. One question comes up persistently: How much secure territory is it worth giving up one amazon for? What about more than one piece? We don't have any definite answers, and can only make one obvious comment: the more turns have been played, the smaller an area needs to be in order to claim it with good confidence. The three problems below explore this question further. You can easily adapt the positions and create follow-up problems. We finish this topic with these wise words of Matt Rudda:

> *Judging whether an area is worth fighting for dominance of is a key skill.*

Material advantage vs. territory size:
The first problem in this set is a puzzle: how many turns do you need to isolate the black piece? This is an interesting mini-game in itself. You can easily create variations, by relocating pieces, changing the available space or adding arrows. Hunts like this occur in actual games, although in less artificial terrain.
In the other two problems, Black has a large, secure territory, and White needs to isolate the lone black queen as quickly as possible.



Problem 4. White to play.



Problem 5. White to play.

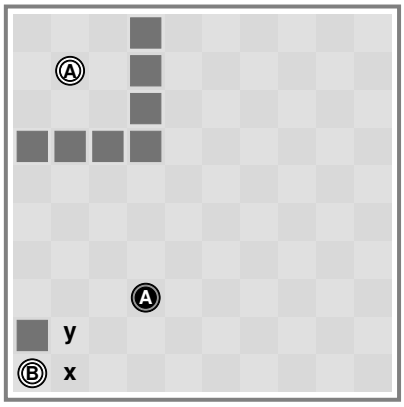
Problem missing!

Mobility and flexibility

Queens have up to eight directions of movement. In her starting position, any queen has five directions available, and only three of those lead towards the interior of the board. Here is a rough classification of amazons according to the number of available directions:

0: isolated/dead, 1,2: threatened, 3,4: restricted, ≥ 5: mobile/free.

The degree of mobility of a queen is not just determined by directions; it is also important how far she can travel. One could simply count the number of possible destinations; for starting amazons, this number is 21. On an empty board, a queen at the boundary has 27 destinations, versus 35 for a queen on one of the four central squares. As the following diagram shows, all those numbers are just guidelines, and they are always trumped by the actual board position:



We consider this irregular position:

Ⓐ can move in all directions but only reach eight squares. This piece is passive and irrelevant for the remainder of the game.

Ⓑ can reach 18 squares but has only two free directions. Black may isolate the corner queen by moving Ⓐ to x, and shooting ■ on y.

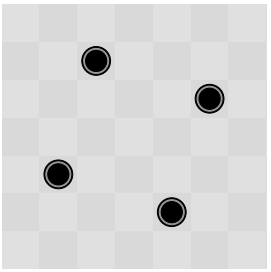
Throughout the game, but especially in the opening, a very important consideration is *flexibility* of one's amazons. Here are some aspects directly related to flexibility:

1. all amazons are mobile and inter-connected, so they can cover each other;
2. every important region of the board can be reached by one, better two, amazons;
3. every amazon, including those defending territories are active.

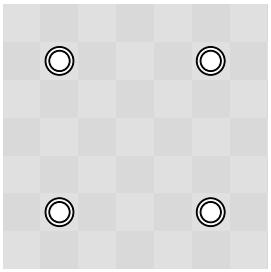
Dually, restricting the opponent's flexibility is equally valuable. Note that flexibility is a concept relating all four amazons, unlike mobility which is a property of individual pieces. We explain some concepts related to flexibility with diagrams:

Place queens on different lines!

Amazons should occupy different ranks, files and diagonals, if possible. The black queens see 8 diagonals, the white ones only 6. It is even worse for rows or columns, with 4 against 2.



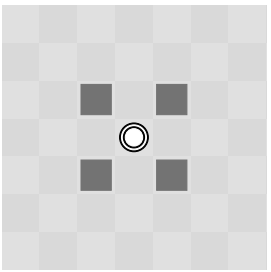
very flexible



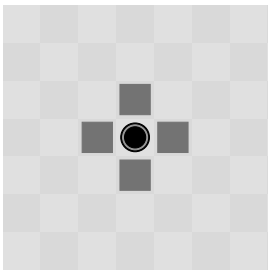
much less flexible

Long diagonals are flexible.

As observed by Matt Rudda in [B], diagonals can be better than rows and columns. This is because moving on a long diagonal covers more space, thus allowing to change areas more quickly.



less flexible

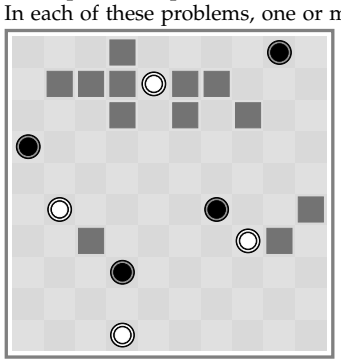


more flexible

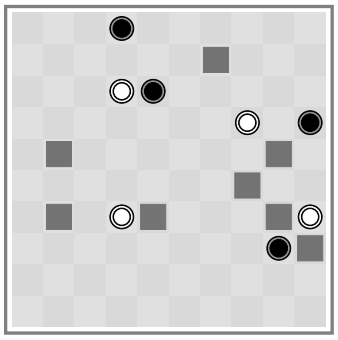
It is often useful to shoot the arrow next to the amazon just moved, whether for building territory or for restricting the mobility of an opposing piece. However, this inevitably reduces the mobility of that amazon, shutting off one direction. Such trade-offs between short and long term gains are typical for AMAZONS.

Ideally, one's own queens should cover the board evenly — crudely speaking, one queen in each quarter. Clustered queens tend to be less flexible and are more prone to be permanently confined to one region, likely rendering one of them superfluous.

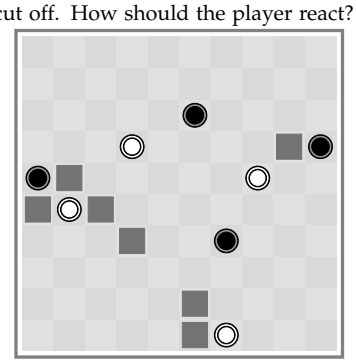
Three positional problems:
In each of these problems, one or more amazons are threatened to become cut off. How should the player react?



Problem 6. White to play.



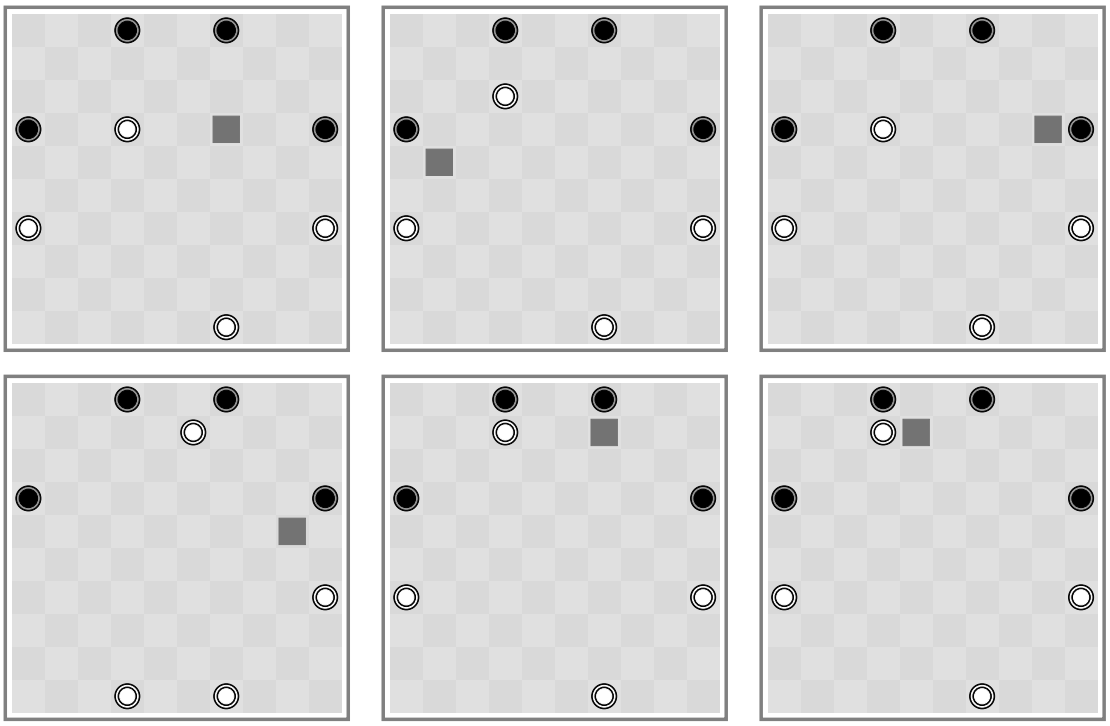
Problem 7. White to play.



Problem 8. Black to play.

Openings

I am not sure if there is any opening theory yet. The following turns have been popular, for obvious positional reasons:



It is worth pointing out that moving the same queen early one is sometimes useful. But such a maneuver comes with a positional loss because it means one amazons sits longer at the boundary than necessary.

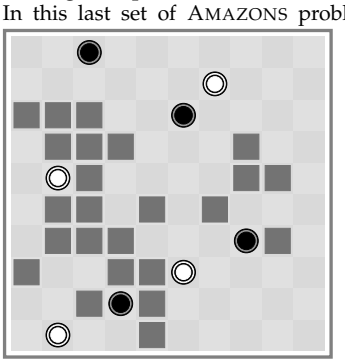
Strategic concepts

Now that we have discussed the endgame, territories, mobility and flexibility, we can mention overarching strategic concepts in AMAZONS. Broadly speaking, throughout a match players have to evaluate *territorial* and *positional* aspects, individually and against each other.

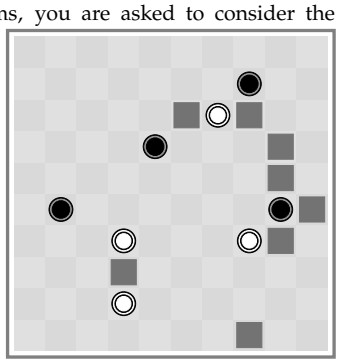
1. constructive territorial move: lay out, build or defend own territory.
2. destructive territorial move: invade or reduce opponent territory.
3. constructive positional move: relocate own piece, increasing mobility or flexibility.
4. destructive positional move: hampering opponent mobility, e.g. isolate a queen.

Ideally, each turn achieves more than just one of the goals. The double move nature of turns often makes this possible.

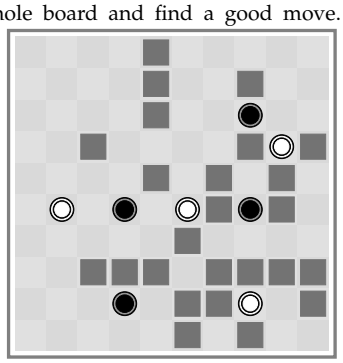
Three global problems:
In this last set of AMAZONS problems, you are asked to consider the whole board and find a good move.



Problem 9. White to play.



Problem 10. White to play.



Problem 11. Black to play.

Historical and other remarks

AMAZONS is one of the few territory scoring games (along with GO the most prominent one). A GO player will notice the similarity but there are a number of important differences: movement instead of placement; isolation of opposing queens but no capture.

What is special about AMAZONS? This is one of the first games to feature double move turns (later examples are SLITHER and MINIMIZE). The game is highly accessible for new players because the movement rules are easy to learn (even if someone does not know CHESS). Each side having only four pieces makes for quick starts (the double moves add a lot of complexity which is not in the way for a new player).

As a board game, AMAZONS players need to balance many competing aspects (constructive and destructive moves), and the game has considerable strategic potential. Moreover, the game is old enough for us to safely say that it has stood the test of time. The existence of a theme adds to the game's appeal.

There are two games which may have (but probably didn't) shaped the creation of AMAZONS. First, the obscure BLOCKADE from 1972 (Frank Ullmann and Tom Werneck) has a mechanism related to arrow-shooting; however, each player has just one piece moving like the Chess king, and neutral markers are placed anywhere; the board is a specially designed graph. And the 1979 pen and paper game CLOSING IN by Sid Sackson plays a little like single-queen AMAZONS where moving and shooting are combined.

Literature

[A] Paul Yearout: *Amazons*, Abstract Games Magazine 16 (Winter 2003), 2 pages.
[B] Matt Rodda: *Strategy in the Game of Amazons*, BoardGameGeek:Amazons:Files (2014), 10 pages https://boardgamegeek.com/filepage/109107/strategy-game-amazons.

Solutions to the problems

Problem 1. White has 20 moves left. Black longest sequence of moves is **h3-g3/j6; g3-e1/c1; e1-f2/e1; e2-g3/e2; g3-i3/g3; i3-j2/f5; j2-i1/h1; i1-j2/i1; j2-h4/j2; h4-h3/h4; h3-i3/h3; i3-j4/i3; ...**missing a single empty square (**d1**). These are altogether 21 turns, so Black wins.

Problem 2. ***b5-b6/b5***. (Problem by Paul Yearout 2003 from [A].)
1... **c6-b7/c6 2**. **b6-a6/b6, h1-d1/c2 3**. **a6-a7/a6, a9-a10/a9 4**. **a7-b8/a7**. Each player now has 29 possible moves left. With White to go first, Black wins.

Problem 3. Analysis: White has 10 (left), 11 (bottom). Black has 1 (bottom left), 3 (top left), 15 (centre right). About the bottom: Black's **d2** will have at some point to retreat into the main area, but she can shoot an arrow at **c1**, forcing White

to cover with **e1-d2**; leading to $8 + 3$ points of White territory.
The top left: if White introduces with **g9-h10**, Black replies with **h9-i9/i10**, and likewise **g8-h8, h9-i9/i8**. In any case, Black will get 4 points in the top right. This makes the open border between **Bd10** and **We8** the most urgent zone. ***d10-e9/f8***.

Problem 4. I am not sure whether 7 or 8 turns are needed. Any input welcome!
If the white pieces start in the corners, then it takes at least one turn longer.

Problem 5. Analysis: Black has 28 secure points on the upper area. The lower area consists of 44 points. Note that 20 turns have been played, and assuming a long game, the expected area for each piece is $(100 - 20 - 8)/8 = 9$. The three black amazons on the upper side contribute $28/3$ points each, which is looking good. The only question is if the remaining black piece can contribute enough: if she gets to make eight turns, then Black will win: these turns will lead to the placement of $2 \cdot 8 = 16$ arrows in the lower area, reducing it to $44 - 16 = 28$ points.
Next, observe that the white pieces on **a4** and **a6** are totally free to move and shoot, whereas the other two amazons will have to shoot at their departure squares.
a3-e3/f3 — this move restricts Black's freedom indirectly but severely. Note how it allows access of all white pieces towards the left.

Problem 6.

Problem 7. The White amazon on **e9** is in danger, as she will be closed into a tiny territory once **e8** is blocked. There is no equivalent counter-threat, so White should definitely rescue this piece. The only line of movement is along the e-file. The most attractive turns seems to be:
e9-e6/h9: ending on a spot with extreme mobility, denying the diagonal to Black's **i10** and threatening to shut her in;
e9-e4/i8:blocking the longer diagonal for Black's **d3** and cutting off the column of **i10**;
e9-e7/b7: puts immediate pressure on Black's **a7**, threatening to confine her to a six point region with **b5-b6/a6** next.

Problem 8. White's amazon **j4** is threatened to become isolated in the next turn (not just by Black **j7-j5/i5** but rather **d10-g7/i5**. One option is to rescue her; however, there is no really good spot along her diagonal of freedom: on **d9**, she has little mobility and is hemmed in by White **d8**; on **h6** or **g7**, she has decent mobility but stands next to White **h7**; this might make **f8** the best option with at least an open file.
An alternative is to set up a counter-threat, and Black **j7** looks like a good target. For example, **h7-i7/i8** which has better mobility but slightly worse position than **h7-i8/i7** (if the Black amazon tries to escape upwards, then White **i8** is better situated to keep her in). A worse move would be **d4-i9/i8**: not only does it fail to capture the Black amazon (she can make trouble by moving to **i7**), it abandons the excellent position in the lower left quadrant.

Problem 9. Black's **a6** is threatened to be isolated in the next turn. Black could rescue her but that would convey a big lower left corner to White. Instead Black plays **g4-e2/c4**. If now White carries out the threat with **d7-b7/a7** (as happened in the game), then the three White amazons **b5, b7, g1** are awkwardly positioned. Black plays **f8-c8/c6**, putting pressure on two White amazons and having the initiave (Black won this game).

Problem 10. Analysis: two White amazons and a Black one populate the lower left, and White will make more points there (approximately $6 : 3$). This also means that White is one queen short in the other, larger area. On the other hand, White's amazons are well-positioned to make territory and it is White's turn.
White played **f3-h3/i3**, immediately claiming the lower right corner. A play in this area is probably urgent because Black's **h4-g3/g2** would have turned the tables, and likely win the game for Black.

Problem 11. White could encircle Black's **i5** with a move such as **d2-g5/h6**. But this neglects the territorial balance, and Black would gain the upper hand with a move like **b5-c5/d5** (slightly better than **b5-d5/c5**), claiming a huge upper left area. Note how, in this development, ill-placed the White amazons are to reduce or invade that region.
The game move is White **d4-d8/e8** (other moves are conceivable as well, I would have preferred **d4-d7/d8** for higher mobility). We evaluate this move from our strategic principles: the White queens are nicely spread on the board, with three of them (**d2, d8, h4** is a good place to make territory.

Problem 12. Black (gzero-bot) played **d2-b2/e2**. I would have taken the corner with **d5-c6/b6**. The game move makes it impossible to secure the corner, though.