What is this, and how can you help?

The following pages contain a short and dense article about a board game. I am writing this (and similar texts for some other games) for basically two reasons:

1. I want to popularise cool board games which are less popular than they should be.

2. There are many accounts (books, online pages etc.) that just give the rules. In order to encourage more people to give them a shot, I'd like to go into a little depth: elementary tactics, problems etc. Hopefully, this helps drawing some future players!

I'm a moderately advanced Go player (1 dan) but not nearly an expert on any of the games I am writing about. Therefore, I will be happy and very grateful for all kinds of feedback. If you think I am way off the mark, please tell me! Remember, the more specific your feedback, the more I can improve the article.

Here are some features that the text is still lacking, but ideally would have:

1. **Problems:** Please have a good look at the problems in the text. Are they well-posed? Do you have ideas for other and/or better problems? (Customarily, problems have unique solutions. I'm not even sure if my current problems have this property.)

2. **More heuristics:** good strategy games have heuristics that allow players to break up the complexity into more manageable pieces. There's not much literature on these games, so I've been starting out in the most simple fashion. If you are using other concepts in your games, please tell me!

3. **Example positions:** if you have encountered a particularly surprising move (by yourself, an opponent, or someone else), feel free to send me the position; most easily as screenshot or LittleGolem link.

I already got some feedback through LG and BGG, and the articles have greatly benefited from that. If you would like to comment, these are the best options:
- right in this thread,
- an email to `dploog@math.fu-berlin.de`. Please mention the game in the subject.

Many thanks for reading this!   June 20, 2019
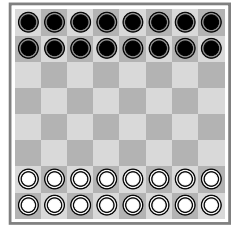
BREAKTHROUGH

BREAKTHROUGH is played on a 8×8 board. Players sit opposite each other, and thus have a home row, and a forwards direction, i.e. towards the opposite home row. Each player has 16 stones which initially occupy the two first rows. White starts.

The active play must carry out precisely one of these actions:
1. **Move** one own stone forward or diagonally forward to an adjacent empty square.
2. **Capture** an opposing stone on one of the two diagonally forward adjacent squares.
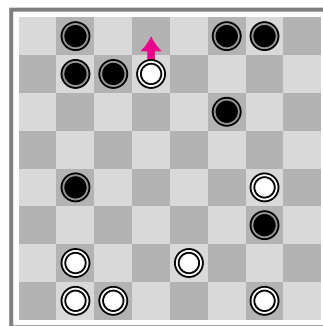
A player **wins** by reaching the opposite home row.
A player **loses** by being unable to move, in particular by having no stones.

## Diagrams explaining the rules
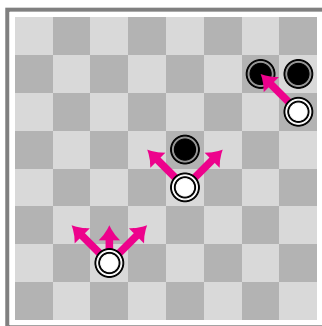
All the possible white moves.

A winning move: White reaches the opposite home row.
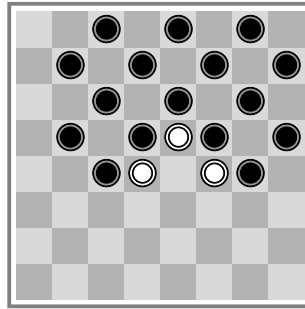
Dan Troyka (2000)

To get a feeling for BREAKTHROUGH, we encourage you to try and solve the following problems right now. If you have trouble, then read the next section on basic concepts, and consider these problems again.
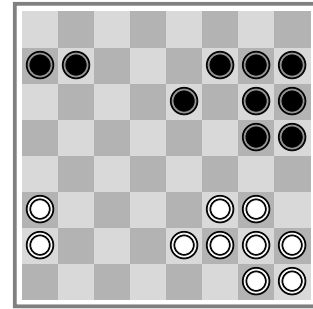

Three easy problems:

Find a winning move for White! Moreover, figure out which white stones could be removed such that the winning combination still works.
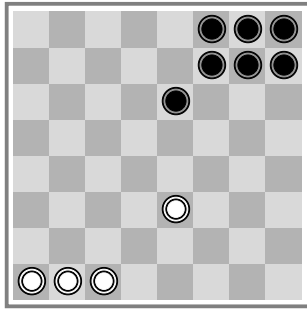
| Problem 1. White to play. | Problem 2. White to play. | Problem 3. White to play. |

Basic concepts

BREAKTHROUGH is a crossing game with exceptionally simple rules. Some immediate properties are: A player always has a valid move because stones can only be blocked by stones of the same colour. Moreover, draws are impossible because of the game's innate forward direction. Since stones are forced to move forwards, each piece has a *mobility cone*:
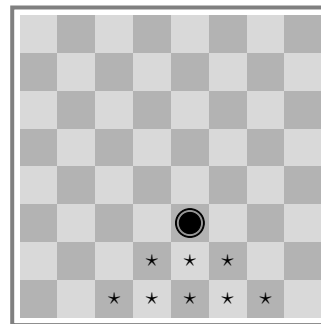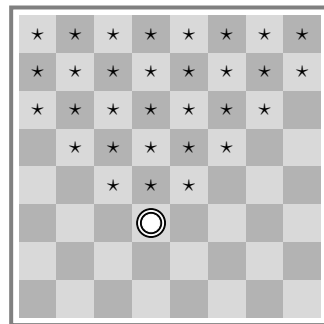
The mobility cones for two pieces: the pieces can only reach squares marked ★.

With every move, the mobility cone of the piece move shrinks. Hence defensive capacities get weaker as a game goes on, and any defense necessarily collapses at some point.

It also provides a straightforward dichotomy for any two pieces of opposite colour: if their mobility cones share at least one square, then one piece can potentially capture the other, and vice versa. Else, the two pieces cannot affect each other. For practical purposes, the more the two mobility cones overlap, the more likely is an encounter. For example, the stones starting on opposite corners can only interact if both move on the long diagonal throughout; hence their encounter is unlikely.

Tactics

Because there are three movement directions but only two for capture, a single stone is unable to stop an opposing stone from breaking through. In particular, if a piece has at most one opposing stone in its mobility cone, then it can reach the goal unhindered. If this happens for the most advanced pieces on both sides, then the game becomes a simple race about whose path is shorter.
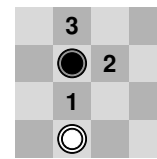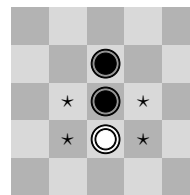
Skirting around
a single stone.

Therefore, at least two stones need to team up in order to form a block. The two most basic shapes are the *horizontal pair* and the *vertical pair*. The next diagram shows how such a pair prevents the single white stone from moving unharmed: all the squares ★ are covered:
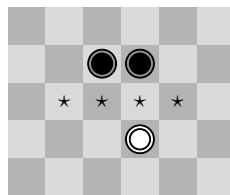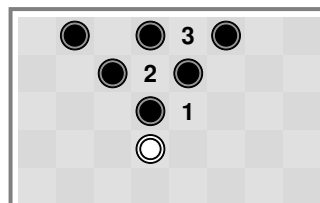
The horizontal pair.   The vertical pair.

By contrast, diagonal structures are very weak at intercepting opposing pieces. In the following position, Black cannot prevent White's solitary stone from breaking through:

White wins unimpeded.

Pieces forming a pair do not protect each other and can be forced to move by an attacker. Bigger formation, *blocks*, are more stable and retain their stopping power even under attack. Consider the following three local positions: in each case, White is attacking a black defensive formation.
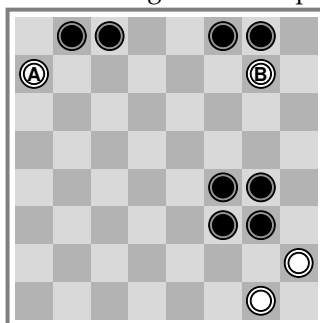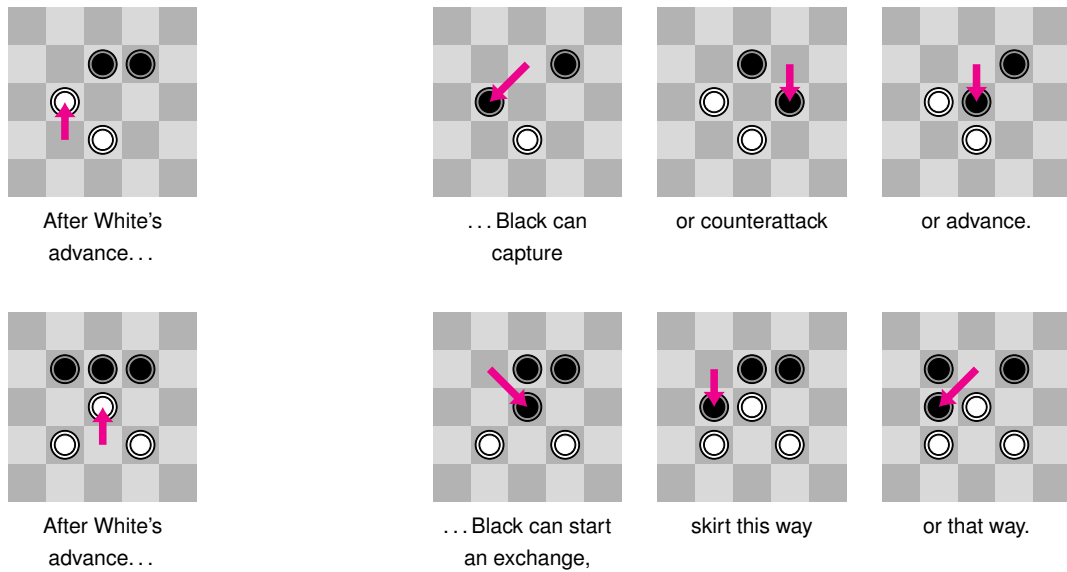
In the top left, Black has to react by capturing Ⓐ. However, this will disrupt Black's pair into two isolated stones.

In the top right, Black again has to capture. But here, Black ends up with a vertical pair.

In the bottom right, Black does not have to react at all: no matter whether White advances or trades pieces, the black block will still be strong enough to prevent either white stone from passing through.

Despite being useless for defensive purposes, *diagonal formations* are fundamental to disrupt enemy blockades. A single piece attacking a defensive structure will be captured immediately, perhaps without further consequences. The defensive breakage may or may not compensate for the loss of the attacking piece. By contrast, if the attacking piece is guarded then the defending player may be forced to break up the blocking formation without capturing.

After White's advance. . .

. . . Black can capture

or counterattack

or advance.

After White's advance. . .

. . . Black can start an exchange,

skirt this way

or that way.

Depending on the surrounding situation, either of the possibilities can be best suited.
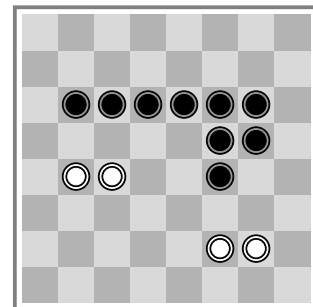
Three attacking problems:

Find a winning move for White! Moreover, figure out which white stones could be removed such that the winning combination still works.
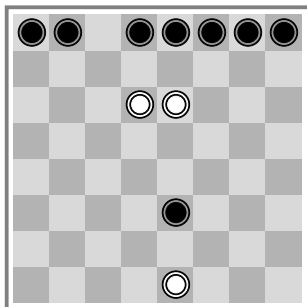
Problem 4. White to play.
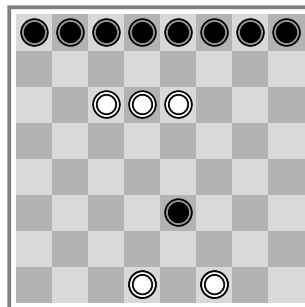

Problem 5. White to play.

Problem 6. White to play.

Strategy

Once the basic tactical principles are understood, strategic considerations will play a fundamental role.

The rules of BREAKTHROUGH promote an aggressive style. Very often, a single attacking piece is able to pin several defenders which is equivalent to a material advantage. Having fewer pieces available to freely move can result in a deadlock, and lead to loss from zugzwang.

Keeping your reinforcement routes open and avoiding being blocked by your own pieces is also important in terms of tempo.
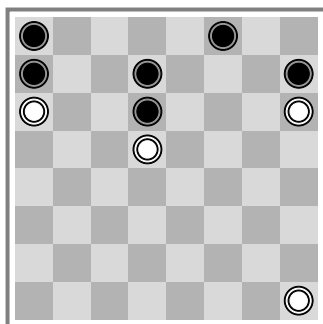
White has three advanced pieces. Neither of them can move without being captured. However, they pin all of Black's pieces: if any black stones moves, then a white stone can break through.

Therefore, by moving the white stone in the corner, Black is put in zugzwang and will quickly lose the game.

Apart from keeping a global material balance, it is also important to keep the local material balance at every stage of the game, since breakthroughs occur when the attacking player is able to accumulate enough offensive power to overwhelm the defenses at their weakest point. Thus the early game is about creating potential for local dominance in some part of the board.

The most obvious type of local material balance is the *left-right balance*. In that sense, there is a tradeoff between keeping your backup forces far enough to maximise their reach and close enough to your other pieces to be helpful in case of a counterattack.

There is, however, another local material balance to take into account: the *checked pattern balance*, referring to the Chessboard pattern. In the right-hand diagram, Black's position seems strong enough to stop White's attack, but it is White's turn, and there are are only two black pieces defending the attack lines of the white pieces. Can you spot the optimal play for White?
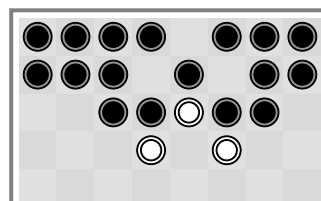
Opening. There are no elaborate opening patterns yet, but the next diagram shows two popular approaches for the first six turns. White's opening leaves the home row intact, striving for defensive flexibility, and builds a large wall on the third row. This wall can be used to gain control of the centre of the board in the next few moves. Black's opening build two solid 3×2 defensive blocks that can resist many attacks without losing strength. However, building those blocks concedes the central rows and the initiative to White.
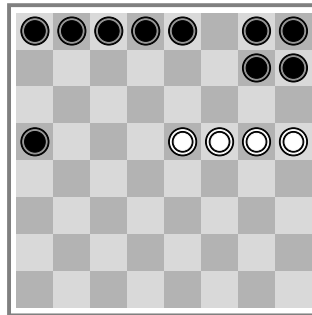
Novice players may feel more comfortable with Black's opening, whereas expert players may prefer White's more aggressive style.
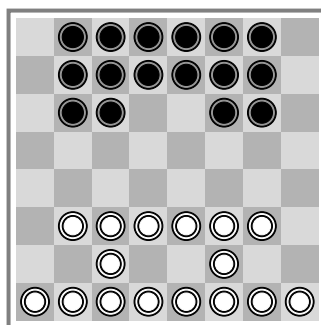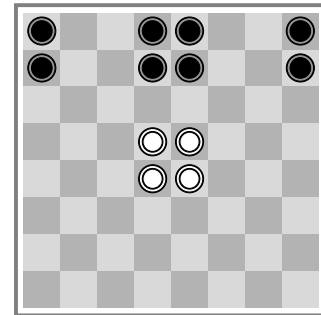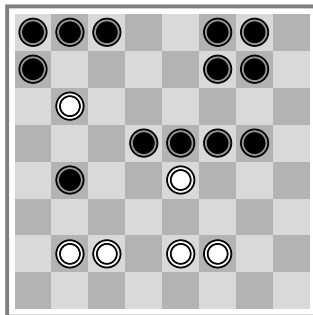
Two opening patterns.

Three problems:
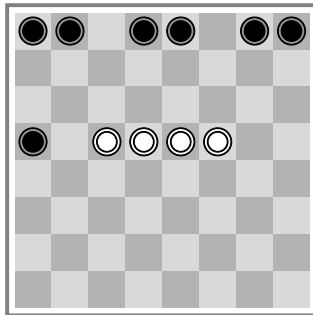
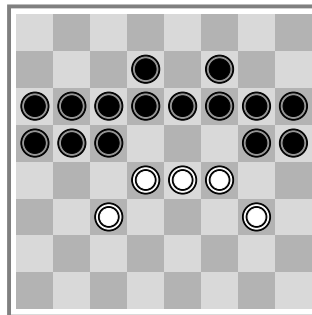

Problem 7. White to play.

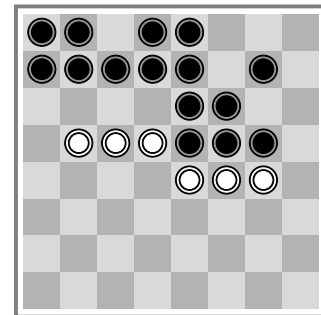
Problem 8. White to play.

Problem 9. White to play.

Three problems:

Problem 10. White to play.

Problem 11. White to play.

Problem 12. White to play.

The crossing goal in other games

There are many games with the win condition of getting one friendly piece across the board. Quite generally, this is a goal with welcome properties leading to positive, race-like encounters. BREAKTHROUGH achieves this with a minimalist set of rules. Without any doubt, similar concepts have been discovered repeatedly, for example in the CHECKERS and the CHESS lineage.

A gentle introduction to CHESS for children is by starting with pawns only. All rules for movement and capture apply, including double moves on starting positions. En passent is optional, and can be introduced, too, although sensibly not for the first few games. The winner is the player to first move a pawn to the opposing home row.

For the purposes of teaching CHESS, a natural next step is to add kings, make mate the goal of the game and allow promotion — that way, the beginner will also learn about queen endgames. By the way, this is how I learned CHESS as a boy.
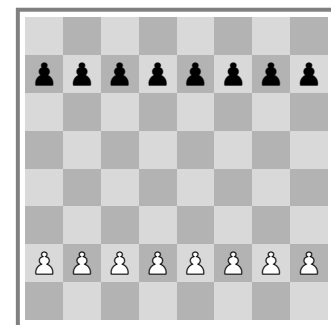
PAWN CHESS

For CHECKERS, one can get shorter and quicker games by replacing the elimination goal with the simpler crossing goal. Again, this reduces complexity due to avoiding promotion. Interest-

ingly, BREAKTHROUGH, pawn-only CHESS and crossing CHECKERS all have a similar asymmetry between movement and capture. I would argue that among these three, and many similar, games, BREAKTHROUGH implements the crossing goal in the most basic fashion.

Solutions to the problems

1. *e3-d4* is the only move. This stone can now reach the goal row in four turns, one turn faster than Black's stone on **e6**. The other three white stones are not needed.
2. *e5-d6* or *e5-f6*. The other two white stones are not needed.
3. White should first play the two forcing moves *f3-e4* and *a3-b4* in either order; Black has to respond with **f7-f6** and **a7-b6**, respectively. Then *a2-b3* will lead a win because all black pieces on the left-hand side are pinned.
4. *d6-d7*. Black needs to capture **e8:d7** but White can recapture **e6:d7** and will break through one turn before Black's can do. The piece on **e1** is not needed.
5. *d6-d7*. The white pieces on **d1** and **f1** are not needed.
6. *b4-b5*. If Black captures, then White will recapture, and is three moves away from breaking through. The white pieces on **f2** and **h2** prevent a faster black breakthrough.
7. *d5-d6, a5-a4; d6-d7!,* e8:d7; e5-e6, d7:e6; f5:e6, a4-a3; e6-d7, a3-a2; d7:e8++. The black piece on the left acts as a clock that advances every time White fails to force a black move. White's **d6-d7** is an example of the level of aggressive play required to win. Finally, the white piece on **c5** does nothing at all in this solution but allows a symmetric solution starting with *e5-e6*.
8. *e5-f6* Not sure this is the only move. Note that Black has only two stones on dark squares in the top right corner, whereas White can attack with all four pieces on dark squares.

Some questions/obervations:

1. What about starting advantage? I haven't seen anyone talk about it.
2. Does the concept of strategic sacrifice exist? Never seen that either.
3. I have the impression that is often better to delay an exchange, i.e. to let the opponent make the first capture. Is this true? Can it be turned into a useful heuristic (when to progress/delay trades)?

Some interesting games from LittleGolem:

https://www.littlegolem.net/jsp/game/game.jsp?gid=1827116 (turn 34!)

https://www.littlegolem.net/jsp/game/game.jsp?gid=1794382 (very centre-oriented)

https://www.littlegolem.net/jsp/game/game.jsp?gid=1437209 (game ends with zero black pieces)