What is this, and how can you help?

The following pages contain a short and dense article about a board game. I am writing this (and similar texts for some other games) for basically two reasons:

1. I want to popularise cool board games which are less popular than they should be.
2. There are many accounts (books, online pages etc.) that just give the rules. In order to encourage more people to give them a shot, I'd like to go into a little depth: elementary tactics, problems etc. Hopefully, this helps drawing some future players!

I'm a moderately advanced Go player (1 dan) but not nearly an expert on any of the games I am writing about. Therefore, I will be happy and very grateful for all kinds of feedback. If you think I am way off the mark, please tell me! Remember, the more specific your feedback, the more I can improve the article.

Here are some features that the text is still lacking, but ideally would have:

1. **Problems:** Please have a good look at the problems in the text. Are they well-posed? Do you have ideas for other and/or better problems? (Customarily, problems have unique solutions. I'm not even sure if my current problems have this property.)
2. **More heuristics:** good strategy games have heuristics that allow players to break up the complexity into more manageable pieces. There's not much literature on these games, so I've been starting out in the most simple fashion. If you are using other concepts in your games, please tell me!
3. **Example positions:** if you have encountered a particularly surprising move (by yourself, an opponent, or someone else), feel free to send me the position; most easily as screenshot or LittleGolem link.

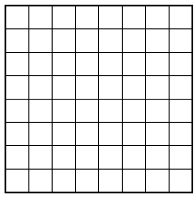
I already got some feedback through LG and BGG, and the articles have greatly benefited from that. If you would like to comment, these are the best options:

- right in this thread,
- an email to `dploog@math.fu-berlin.de`. Please mention the game in the subject.

Many thanks for reading this! May 16, 2019

SYMPLE

9 × 9



40 × ○ ●

The 9 × 9 board is initially empty. White goes first.

In these rules, 'adjacent' means 'orthogonally connected'. A *chain* is a maximal connected set of stones of one colour. *Extending* a chain means to place one stone of the chain's colour on a vacant point which is adjacent to at least one stone of the chain. Players only place stones of their own colour, and always on vacant points.

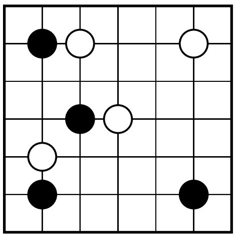
On a turn, a player must either
- **plant**: place a stone on a vacant point having no adjacent stones of the same colour, *or*
- **grow**: extend each chain of the player's colour by exactly one stone, if possible.

Moreover, as long as no player has grown yet, Black may first grow and then plant in a single turn.

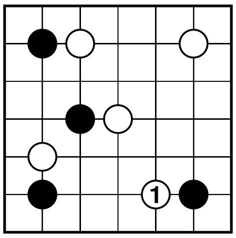
The game **ends** if the board is full. At that point, the *score* of each player is the number of stones placed minus 4 points for each chain of that colour. The winner is the player with the higher score.

Diagrams on rules

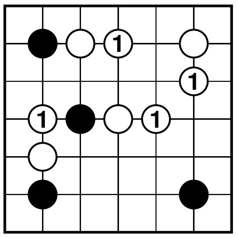
SYMPLE is eminently scalable, and we give illustrations of the rules on 7 × 7 boards.



White to play:

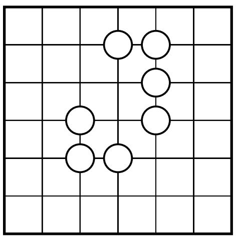


*Either **plant** one stone.*

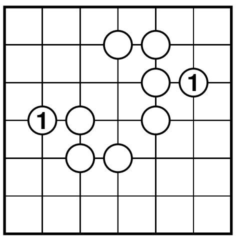


*Or **grow** each chain.*

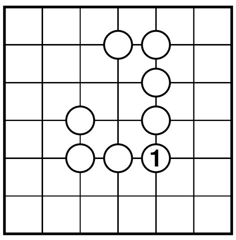
The next diagrams show some valid growth moves for the two white chains shown. Note that in the right-hand diagram, each of the new stones extends just one chain of the starting diagram.



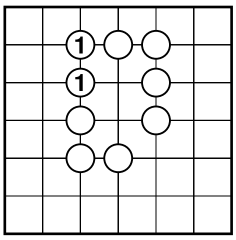
White to grow.



Separate growth.



Unification.



Touching unification.

Christian Freeling & Benedikt Rosenau (2010)

Actual gameplay

SYMPLE can be conveniently played online, at mindsports.nl and using Stephen Taverner's AiAi. Playing in this fashion also keeps track of current scores.
When playing with real stones, it can become hard to check which chains have been grown and which have not. A feasible solution is to temporarily place stones of some other kind (for instance, other colour or other type — these could be pennies!) everywhere chains shall grow. When satisfied, replace these with proper stones.
Another issue is keeping track of points, i.e. the difference between Black and White stones on the board. This should be tracked after Black moves (because that is much less swingy than checking after every turn).

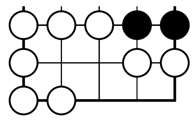
Plant vs. grow

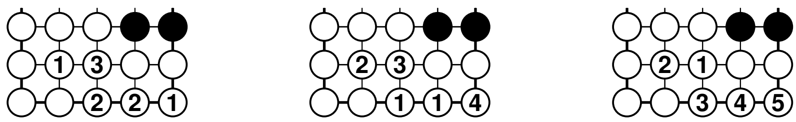
In SYMPLE, the goal is to get as many stones on the board as possible in as few chains as can be. This dilemma carries the game to a large extent: having more chains is good for further growth, hence points, but it becomes a liability due to the score penalty for each chain. Because players have to move, running out of growth options means that a player is forced to plant inside an opposite territory. The new stone will then likely not out-grow the chain penalty, and thus incur a loss. Therefore, many games have a *cold phase*, where players try to delay or avoid negative moves. This leads to a nuanced middle game. Roughly speaking, SYMPLE games proceed like this:

1. Opening: planting moves exclusively.
2. Midgame: from first growth until territories are traced out. Border battles and invasions.
3. Endgame: territory filling and connections. Cold phases.

Growth order matters

It can happen that a particular choice of growth prevents another chain from growing. For example, White can fill the adjacent corner area in three, four or five turns. Filling a region as slowly as possible can be important, especially in a localised shape such as this: during a cold phase both players strive to avoid having to plant a new stone in an opposite territory.





The slowest approach is to connect as early as possible. In a bigger position, matters are more subtle: regarding points on the whole board, delayed connections are preferable.

Second mover compensation

Black has the privilege of growing *and* planting, but only if neither player has grown before. This is a compensation for going second, and creates some tension in its own right: in principle, both sides want to delay growth and start planting. But if White does that for too long, then Black will carry out the double action. This provides incentive for White to grow sooner than otherwise intended, which in turn does the same for Black!
As an example, it is not a good idea for Black to use the double action privilige on turn 2. By doing so, Black has placed one more stone but now White can plant without having to worry about a later growth & plant of Black.

The meaning of the penalty P

The original rules don't specify the chain penalty for scoring. We provide $P = 4$ as a good starting value on the 9×9 board. On the 13×13 board, we suggest $P = 6$, and on the 19×19 board, start out with $P = 10$. The designers consider the choice of P as part of the game. A choice of an even number for P on an odd-sized square board prevents draws. As so often, it is interesting to ponder extreme cases:

$P = 0$:	In this case, the only goal is to maximise stone count at the end of the game.
$P = 169$:	Win by smaller number of chains; equality is broken by number of stones.
$P = -169$:	Win by larger number of chains; equality is broken by number of stones.

Curiously, none of these extreme variants leads to degenerate, entirely trivial games. Without going into details, this follows from the plant/grow protocol, cutting and connections, and the resulting cold phase (if $P \neq 0$). That forebodes well for the real game with more modest penalties!

Optimising agriculture

Let us focus on the stone-maximising part. Obviously, any optimal procedure would start with planting a number of stones, and then keep on growing.
Assume that we first take n turns planting stones, followed by m turns of growing. Then we end up with $n(m+1)$ stones after $n+m$ turns. Also assume that we play on a 13×13 board and want to reach at least 80 stones, which is roughly half of it. (The bigger numbers show better what is going on.) Here are some ways how to do that:

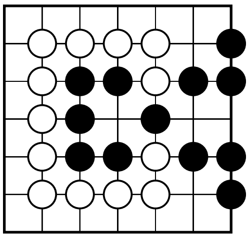
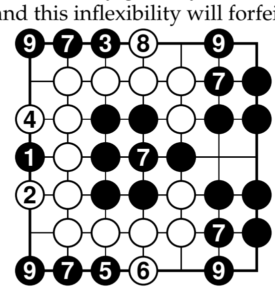
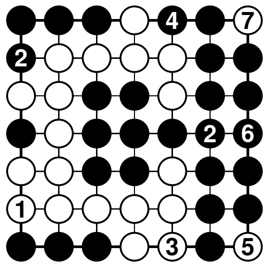
first plant:	n	—	4	5	6	7	8	9	10	11	12	13	14	15	16
then grow:	m	—	19	15	13	11	9	8	7	7	6	6	5	5	4
stones placed:	$n(m+1)$	—	80	80	84	84	80	81	80	88	84	91	84	90	80
total turns:	$n+m$	—	23	20	19	18	17	17	17	18	18	19	19	20	20

We see that at least 17 turns are necessary to cover an area of 80 points, and that there are three possibilities for this, planting 8, 9 or 10 turns before switching to growth. It seems reasonable that nearby values, such as planting for 7, 11 or 12 turns should also be relevant for good play.

Digression: In general, an optimal way to populate an area of a points takes $n \approx \sqrt{a}$ plantings and $m \approx \sqrt{a} - 1$ growths. As the above example $a = 80$ shows, due to rounding, nearby n values can also work. ($n(m+1) = a$, so $m = a/n - 1$. We want to minimise the function $f(n) = n + \frac{a}{n} - 1$. Its derivative is $f'(n) = 1 - a/n^2$ which becomes zero for $n = \sqrt{a}$. Then $m = \frac{a}{n} - 1 = \sqrt{a} - 1$.)

Inflexibility from too early connection

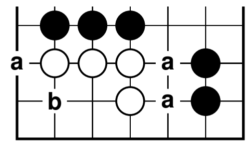
In the following position, it looks as if White has staked out a comfortably large territory. However, the white stones form a single chain, and this means that White can only grow by one stone in each turn. Thus White necessarily reacts very slowly to Black's repeated invasions, and this inflexibility will forfeit the game.

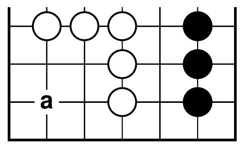
This is just one possible development. It ends with $28 - 4 \cdot 4 = 12$ points for Black and $21 - 3 \cdot 4 = 9$ points for White.
Not sure this holds for all developments.
Also not sure this is the best way to explain inflexibility. However, the inflexibility effect is real and should be covered.

Defensive moves

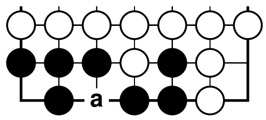
During a SYMPLE game, invasions are important: if properly timed, they will reduce opposing territory, and gain time in cold phases. By the same token, defensive moves can be valuable.



Defensive growth: In this position, White has staked out a corner territory. Growing at any of the points marked *a* are natural moves either extending the white corner or walking towards black chains. By contrast, the inward move *b* aims at preventing an invasion or at least making it less profitable. Compare this position with a black invasion at *b*.



Defensive planting: In the position shown here, White planting at *a* can be a useful turn. While this is slow, it prevents a black invasion at the same spot (which would grow to a sizable chain), and further growing moves will help cutting the corner into small regions faster.

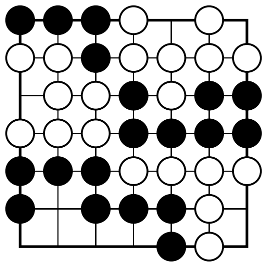


Delaying cut: In this position, a White cut at *a* is a net gain of 2 points: it incurs the penalty for the one-stone chain just placed, but it also prevents the black connection. The gain is marginal, especially since White is forfeiting growth. But this may just the be goal during a cold phase.

Analysis of an endgame position

Now we look closely at a SYMPLE position in a final stage. This will exhibit the following concepts, although on a tiny scale: cold phase, invasions, defensive planting. The combinatorial complexity becomes enormous for more open positions, and one has to rely on heuristics.

We will analyse this position, starting with most naïve approach: both sides just keep growing. In a next step, one has to consider invasions (offensive plantings), counter-invasions, and defensive moves (both growth and planting). This holds for endgame positions. As usual for board games, the opening and midgame are even less scripted.

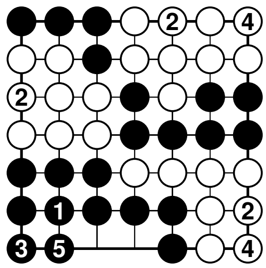


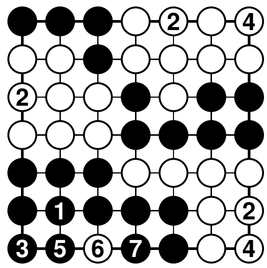
Black to play, $P = 4$

Consider the position on the left.

Chain sizes: $7 + 7 + 6 = 20$ for White and $4 + 7 + 8 = 19$ for Black.

Each colour has precisely three chains which cannot connect anywhere. Moreover, both sides enclose 5 points in their areas but — and this is an important difference — these make up a single black territory while they are distributed over three white areas.

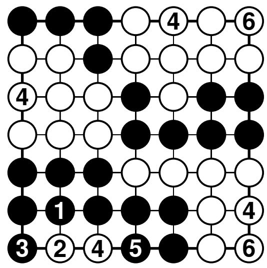




Black and White just keep growing.

The first and most straight-forward case to test is mutual growth. However, with ❺, White has lost the cold war and is forced to plant ⑥ inside Black's lower left area. The game ends with a Black win:

Black: $23 - 3 \cdot 4 = 11$ points,
White: $26 - 4 \cdot 4 = 10$ points.

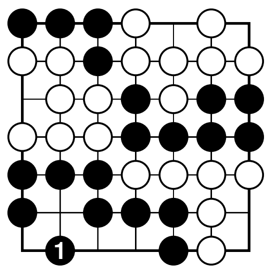


Black grows and White invades immediately.

So White must do better: being forced to plant inside Black's territory anyway, it is best to do it immediately and make the invading chain as large as possible. This secures a one point win for White:

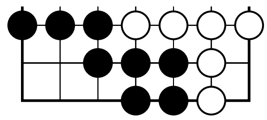
Black: $22 - 3 \cdot 4 = 10$ points,
White: $27 - 4 \cdot 4 = 11$ points.

Instead of ②, White can also invade on the point to its right.

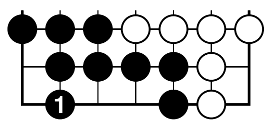


Black's correct move: defensive planting.

Going back to the original position, we see that ❶ should be planted inside the Black territory. Now White will not be able to grow the invading stone, and hence lose. It is left to the reader to check what happens if White keeps on planting inside Black's corner.
In an actual game, on a bigger board, it is dependent on circumstances whether and when a defensive planting move like ❶ is appropriate.

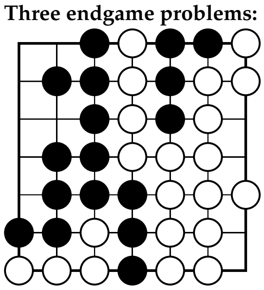


This solution hinges on the shape of Black's lower left corner territory. With the 5-point area shown here, no Black placement could have prevented White's invasion and subsequent growth. In this position, Black will lose, assuming that White plays optimally.

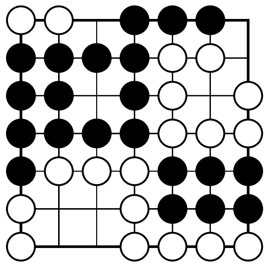


And if the lower left region had a more favourable shape, then a simple growth move would have sufficed. This indicates that early defensive growths can prove very beneficial in the long run: aim for flat, linear regions rather than compact, square-shaped ones.

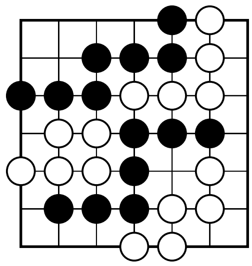
Three endgame problems: *White to play.*



Problem 1.



Problem 2.



Problem 3.

Solutions to the problems

Problem 1. ***a5***. Black should block on the upper side, **a6**. After a double growth of White and single growth by Black, White has to plant one last stone into the black region. However, White wins with $21 + 4 + 2 - 3 \cdot 4 = 15$ points, to Black's $18 + 4 - 2 \cdot 4 = 14$.

Problem 2. ***c7, b1, b2, g6***, the crucial point is the touching unification in the bottom left corner. This prevents a successful (i.e. eventually two-stone) Black invasion. After final growth, Black has to invade once, ending as $3+9+13-3 \cdot 4 = 13$ points, more than Black's $17 + 6 + 1 - 3 \cdot 4 = 12$.

Problem 3. Growing ***a2, c1, g5*** ensures that White will be able to connect two of the three chains. This will win. An invasion **b7** (the best spot in the upper left corner) will lose: Black will grow **c8, g4**, and afterwards White is stuck with four chains whereas Black only has two.

Todo

1. What is an invasion worth? (Territorial break-even at size $P/2$, plus potential gain in cold phase.) Opportunity cost of the missed growth?
2. Cold phase management. (I have no clue on this.)
3. More problems. But I am not sure I can pose anything better than micro-endgame problems.
4. I'm okay even with just four pages on SYMPLE: the movement protocol should be explained, and some of its consequences, too. Perhaps someone can help with deeper heuristics and better problems.