Foreword

These notes had two purposes:

(a) To make various corrections and amendments to our earlier sets of notes [NFS], [Addendum to NFS], [MOI],

(b) To extend the theory developed in [NFS] to all premice in the sense of the early chapters of [NFS] (even premice with "superstrong extenders"). In particular we wanted to extend the results of §7 and §8 of [NFS] to these structures.

(This explains the disparate lengths of the various sections.)

In order to carry out (b) we had to make two changes, which made our theory more like Steel's. First of all we introduced a stronger initial segment condition which was somewhat like Steel's condition. (In I we show, however, that this condition cannot be essentially weakened.) In addition, we introduced k-ultraproducts and k-iterations, in addition to the Σ^* and Σ_0-ultraproducts used previously. It would seem that we introduced them for much the same reasons that Steel did, though the difficulties which impelled their usage occured later in our theory than in Steel's (not until superstrong extenders). The theory of k-iterations was developed imperfectly in II and is redone in the correction to II. Later, however, Martin Zeman showed how to get all of the results given here without the use of k-ultraproducts. I believe that Zeman has the superior approach. It requires doing things in a different order, however, forcing an amalgamation of §7 and §8 of [NFS]. We sketch Zeman's proof in a final addendum.

Corrections and Remarks

These notes contain corrections and other additions to our handwritten notes:

[NFS] A New Fine Structure for Higher Core Models

[ANFS] Addendum to [NFS]

[MOI] More on Iterability

We also refer to:

[MS] Mitchell, Steel <u>Fine Structure and Iteration Trees</u>

[S] Steel <u>The Core Model Iterability Problem</u>

In I we deal with an embarrassing fact which was pointed out by Itay Neeman. The initial segment condition used in the definition of "premouse" does not appear to have the requisite preservation properties — e.g. preservation under iterations. We therefore formulate a stronger initial segment condition (IS) and show that it does have these properties. Strengthening the initial segment condition of course brings with it the danger of unduly restricting the class of mice. However, we show that any initial segment which satisfies certain minimal adequacy conditions will imply that MS holds for mice. Hence we have avoided that danger.

In II we prove the solidity lemma for mice in full generality. In §7 of [NFS] it was originally proven for 1-small mice. In the appendix to §7 we tried to indicate a more general proof, which, however is insufficient. The problem arises in mice which have superstrong extenders. In order to handle the difficulty we had to introduce "k-iterations" in place of "$*$-iterations". (k-iterations were in fact the tool used by Steel to prove solidity).

In III we perform the same service for the theorems of [NFS] §8. As pointed out in the appendix to §8 the condensation lemmas were proven only under a fairly restrictive assumption. We now reformulate and prove them in full generality. In order to handle the possibility of superstrong extenders we had to reformulate the main condensation lemma (Lemma 4) by adding a further disjunctive clause to the conclusion. The new proof again required k-iterations.

In V we make use of the generalized condensation lemma of IV to show that the work of Schimmerling and Zeman on $\Box$ is optimal. We consider a model L^E, all of whose proper segments are weak mice (in the sense of I.) Let λ be a cardinal in L^E. Set $D =$ = the set of $\nu \in (\lambda, \lambda^+)$ which index a superstrong extender (i.e. the length of E_ν is λ). We show that $\square_\lambda$ fails if D is stationary. By the work of Schimmerling and Zeman, $\square_\lambda$ holds if D is not stationary.

In VI we indulge in science fiction. We suppose the "ultimate" K^c model, whose construction employs all possible premice in the sense of [NFS], had been built up to a "large" cardinal Θ (e.g. measurable). We examine the consequences of a failure of the "cheap covering lemma" for K^c, which says that the set $\{\tau < \Theta \mid \tau^{+K^c} < \tau^+\}$ is "small" (e.g. of measure 0). If this fails we show that a strong axiom of infinity holds on a "large" set. We also prove outright that the set $\{\tau < \Theta \mid cf(\tau^{+K^c}) < \tau\}$ is "small". (We use "subtle" in place of "measurable", but the proofs are virtually the same.)

VII was previously appended to our notes [MOI].