§ 3.4 The Core Model

§ 3.4.1 Strong mice

Def A mouse $N = \langle J_\alpha^E, E_\alpha \rangle$ is strong iff whenever M is a premouse s.t. $M|\alpha = N$ and M is iterable above α, then M is a mouse and $N = (\mathrm{core}(M)|\alpha)$.

Lemma 1 Let $N = \langle J_\alpha^E, E_\alpha \rangle$ be a mouse. The following are equivalent:

(a) N is strong

(b) There is a universal weasel W s.t. $N = W|\alpha$.

proof.

(a) $\rightarrow$ (b) Define a hierarchy W_ν $(\alpha \leq \nu < \theta \leq \infty)$ with $W_\nu = \langle J_{\gamma_\nu}^{E^{W_\nu}}, E_{\gamma_\nu}^{W_\nu} \rangle$ $(\gamma_\nu \leq \nu)$ as follows:

$W_0 = N$. Let W_ν be defined. If W_ν is a mouse and $N = W_\nu | \alpha = \mathrm{core}(W_\nu)|\alpha$, set:

$$W_{\nu+1} = \langle J_{\gamma+1}^E, \emptyset \rangle, \text{ where}$$

$$\mathrm{core}(W_\nu) = \langle J_\gamma^E, E_\gamma \rangle.$$

Otherwise $W_{\nu+1}$ is undefined.

If W_ν is defined for $\nu < \lambda$ (where $\lambda > \alpha$ is a limit ordinal), we define W_λ exactly as in the def. of the canonical ω-complete hierarchy in §3.1.

[We call this hierarchy $W_\nu = W_\nu[N]$ the <u>canonical</u> ω-<u>complete</u> <u>hierarchy</u> <u>over</u> N].

We verify by induction on $\tau \geq \alpha$ that W_τ is defined. $\tau = \alpha$ is immediate. For limit τ, the proof is as before. Now let $\tau = \nu + 1$. Then W_ν is iterable above α exactly as before. But $W_\nu | \alpha = N$. Hence by strongness W_ν is a mouse and $\mathrm{core}(W_\nu) | \alpha = N$.

Define $W_\infty = W_\infty[N]$ as before. It follows as before that W_∞ is universal and $W_\infty | \alpha = N$.

QED (a) $\to$ (b)

(b) $\to$ (a) Let M s.t. $M | \alpha = N$ be iterable above α. Coiterate M, W to M', W'. By universality, M' is an initial segment of W' and a simple iterate of M. Hence M is a mouse. It remains to show:

<u>Claim</u> $M|\alpha = \mathrm{core}(M)|\alpha$.

If $\rho^\omega_M \geq \alpha$ there is nothing to prove. Assume $\rho^\omega_M < \alpha$. Let $\bar{M} = \mathrm{core}(M)$, $\rho = \rho^\omega_M$. Then $\bar{M}|\rho = M|\rho = W|\rho$. Let $\langle \bar{M}_\xi \mid \xi \leq \theta \rangle$, $\langle W_\xi \mid \xi \leq \theta \rangle$ be the coiteration of $\bar{M}$, W. Then $\theta < \infty$ and the $\bar{M}$ side is simple by universality.

<u>Subclaim</u> $\bar{M} = \bar{M}_\theta$

prf.

If not, $\bar{M}_\theta$ cannot be a proper segment of W_θ. Hence $\bar{M}_\theta = W_\theta$. Let $\langle \nu_\xi, \alpha_\xi \rangle$ be the indices of the W-side and let ξ be maximal s.t. $\omega\alpha_\xi \in W_\theta$. Then $W_\xi|\alpha_\xi$ is sound and W_θ is an iterate of $W_\xi|\alpha_\xi$ above $\rho^\omega_{W_\xi|\alpha_\xi} = \rho^\omega_{W_\theta} = \rho$. Hence $W_\xi|\alpha_\xi = \mathrm{core}(W_\theta) = \bar{M}$. Hence $\bar{M}_\xi \neq \bar{M}$. Hence there is a least $\varsigma < \xi$ s.t. $\overline{M}_{\varsigma+1} \neq \overline{M}$. Then

$$E^{\overline{M}_\xi}_{\nu_\varsigma} = E^{\overline{M}_{\varsigma+1}}_{\nu_\varsigma} = \emptyset \text{ and } E^{W_\xi}_{\nu_\varsigma} = E^{\overline{M}}_{\nu_\varsigma} \neq \emptyset.$$

Hence $\nu_\xi \leq \nu_\varsigma$ where $\varsigma < \xi$.

Contradiction! QED (Subclaim)

We now prove the Claim. Suppose not. Let ν be least s.t. $E^{\overline{M}}_\nu \neq E^M_\nu$. Then $\nu \leq \alpha$ + ν is least s.t. $E^{\overline{M}}_\nu \neq E^W_\nu$, since $M|\alpha = W|\alpha = N$. But $E^{\overline{M}}_\nu \neq \emptyset$ and $E^M_\nu = E^W_\nu = \emptyset$, since M is a simple iterate of $\overline{M}$. But this contradicts the subclaim.

QED (Lemma 1)

As a corollary of the proof:

<u>Lemma 1.1</u> N is strong iff the canonical ω-complete weasel $W_\infty[N]$ over N exists.

We now define a weasel $K = J_\infty^E$ which we do not yet know to exist.

<u>Def</u> A hierarchy $K_\nu = \langle J_\nu^E, E_\nu \rangle$ of strong mice is defined by:

$K_0 = \langle \emptyset, \emptyset \rangle$

$K_{\nu+1} = \langle J_{\nu+1}^E, \emptyset \rangle$ where $K_\nu = \langle J_\nu^E, E_\nu \rangle$

($K_{\nu+1}$ is easily seen to be strong if K_ν is).

For limit λ let $J_\lambda^E = \bigcup_{\nu<\lambda} J_\nu^E$. If $\langle J_\lambda^E, \emptyset \rangle$ is strong and there is no $F \neq \emptyset$ s.t. $\langle J_\lambda^E, F \rangle$ is strong, set: $K_\lambda = \langle J_\lambda^E, \emptyset \rangle$.

If there is a unique $F \neq \emptyset$ s.t. $\langle J_\lambda^E, F \rangle$ is strong, set:
$K_\lambda = \langle J_\lambda^E, F \rangle$.

Otherwise K_λ is undefined

If K_ν is defined for all $\nu < \infty$, we define the <u>core model</u>:
$K = J_\infty^E = \bigcup_\nu J_\nu^E$.

Our aim is to show that K exists, is universal, and that every universal weasel is a simple iterate of K (by an iteration of length $\leq \infty$). In §3.4.2 we prove these facts under the assumption that O^{2} exists. The remainder of the paper will then deal with the more difficult (but more interesting) case: $\neg O^{2}$.

[Note that K, if it exists, is universal since E_{ν}^{K}, being unique, is chosen as ω-complete wherever possible. Hence the proof which showed the canonical ω-complete model to be universal shows K to be so.]

§ 3.42 If 0^1 exists.

Throughout this section assume that 0^1 exists. Recall that every 1-mouse $N = \langle J_\alpha^E, E_\alpha, E_{\alpha+1} \rangle$ is a simple iterate of 0^1; hence $\rho_N^1 = \rho_{0^1}^1 = 1$ and $*$-iterations of 1-mice are the same thing as ordinary iterations.

Def Let N be an 1-mouse. Let N^i $(i < \infty)$ be the iterates obtained by iterating the top measure — i.e. the iteration indices are $\nu_i = \alpha_i + 1$, α_i where $N^i = \langle J_{\alpha_i}^{E^{N^i}}, E_{\alpha_i}^{N^i}, E_{\alpha_i+1}^{N^i} \rangle$.

It is clear that $N^i | \alpha_i = N^j | \alpha_i$ for $i \leq j$ (taking $\langle J_\alpha^E, E_\alpha, E_{\alpha+1} \rangle | \alpha$ to mean: $\langle J_\alpha^E, E_\alpha \rangle$).

Set: $N^\infty = \bigcup_i J_{\alpha_i}^{E^{N^i}}$. Then $N^\infty | \alpha_i = N^i | \alpha_i$ for $i < \infty$.

An obvious fact is:

Fact $(N^i)^j = N^{i+j}$ $(i<\infty,\ j\leq\infty)$.
Hence $N^\infty = (N^i)^\infty$ $(i<\infty)$.

Def $\tilde{K} = (0^1)^\infty$.

Our aim is to prove: $\tilde{K} = K$.

. . . . . .

Def Let $\sigma : \bar{N} \xrightarrow[\Sigma_0]{} N$ cofinally, where $\bar{N}, N$ are 1-mice. By induction on i define:

$$\sigma^i : \bar{N}^i \xrightarrow[\Sigma_0]{} N^i \text{ cofinally}$$

s.t.

(a) $\sigma^i \pi_{\bar{N}^h \bar{N}^i} = \pi_{N^h N^i} \sigma^h$ $(h\leq i<\infty)$

(b) $\sigma^i \upharpoonright \bar{N}^h = \sigma^h$ (")

Set: $\bar{\pi}_{hi} = \pi_{\bar{N}^h, \bar{N}^i}$, $\pi_{hi} = \pi_{N^h, N^i}$.
The inductive definition reads:

$\sigma^0 = \sigma$

$\sigma^{i+1}(\bar{\pi}_{i,i+1}(f)(\bar{\kappa}_i)) = \pi_{i,i+1}\sigma^i(f)(\kappa_i)$

$\sigma^\lambda \bar{\pi}_{i\lambda} = \pi_{i\lambda}\sigma^i$ for $\lim(\lambda)$, $i<\lambda$.

The inductive verification of (a), (b) is straightforward

Def $\sigma^\infty = \bigcup_i \sigma^i$.

Hence $\sigma^\infty : \bar{N}^\infty \xrightarrow[\Sigma_0]{} N^\infty$.

Lemma 1 Let $\sigma : \bar{N} \xrightarrow[E_\nu]{} N$, $\omega\nu \leq On \cap \bar{N}$, then $\sigma^i : \bar{N}^i \xrightarrow[E_\nu]{} N^i$ for $i \leq \infty$.

prf. By ind. on i.

(Recall that $E_\nu^{\bar{N}} = E_\nu^{\bar{N}^i}$ $(i \leq \infty)$).

An obvious converse is:

Lemma 2 Let $\tilde{\sigma} : \bar{N}^\infty \xrightarrow[E_\nu]{} Q$, where $\nu \leq On \cap \bar{N}$. Set $\sigma = \tilde{\sigma} \upharpoonright \bar{N}$. Let $N = \langle J_\alpha^E, E_\alpha, E_{\alpha+1} \rangle$ be defined by: $N|\alpha = Q|\alpha$ and $\sigma : \bar{N} \xrightarrow[\Sigma_0]{} N$ cofinally. Then $\sigma : \bar{N} \xrightarrow[E_\nu]{} N$. (Hence $\tilde{\sigma} = \sigma^\infty$ and $Q = N^\infty$, since $\sigma^\infty : \bar{N}^\infty \xrightarrow[E_\nu]{} N^\infty$).

prf. Straightforward.

As straightforward corollaries of these lemmas we get:

Cor 3 Let N be an iterate of $\bar{N}$ by an iteration $\langle \bar{N}_i \rangle$ with indices $\nu_i \leq \alpha_i = On \cap \bar{N}_i$. Let $\sigma = \pi_{\bar{N}N}$. Then N^i is an iterate of $\bar{N}^i$ by an iteration with the same indices and iteration map $\sigma^i = \pi_{\bar{N}^i N^i}$ $(i \leq \infty)$.

Cor 4 Let Q be an iterate of $\bar{Q} = \bar{N}^\infty$ by an iteration $\langle \bar{Q}_i \rangle$ with indices $\nu_i \leq \pi_{\bar{Q}\bar{Q}_i}(On \cap \bar{N})$. Set $\sigma = \pi_{\bar{Q}Q} \upharpoonright \bar{N}$. Let $\alpha = \pi_{\bar{Q}Q}(On \cap \bar{N})$ and let $N = \langle J_\alpha^E, E_\alpha, E_{\alpha+1} \rangle$ be defined by $N|\alpha = Q|\alpha$ and $\sigma : \bar{N} \xrightarrow{\Sigma_0} N$ cofinally. Then N is an iterate of $\bar{N}$ by an iteration with the same indices and iteration map $\sigma = \pi_{\bar{N}N}$. (Hence $Q = N^\infty$ and $\pi_{\bar{Q}Q} = \sigma^\infty$).

We use these lemmas to prove an important structural relationship between $\tilde{K}$ and $O^\sharp$:

<u>Lemma 5</u> Let $\langle N_i \mid i \leq \theta \rangle$ be a simple normal iteration of $O^\sharp$ with indices ν_i. Let K_i $(i \leq \theta)$ be the iteration of $\tilde{K}$ with the same indices. Then $N_i|\alpha_i = K_i|\alpha_i$ $(i \leq \theta)$, where $\alpha_i = On \cap N_i$

(Note that $E^{K_i}_{\alpha_i+1} = \phi$, whereas the lemma tells us that $E^{K_i}_{\nu_i} = E^{N_i}_{\nu_i}$ for $\nu_i \leq \alpha_i$).

proof.

Let $M^j = \langle J^{E^j}_{\beta_j}, E^j_{\beta_j}, E^j_{\beta_j+1} \rangle$ be the iterates of M by the top measure $(j < \infty)$. Then $M^\infty = \tilde{K}$.

Since θ is arbitrary it suffices to show:

<u>Claim</u> $N_\theta|\alpha_\theta = K_\theta|\alpha_\theta$.

Let $k = k_\theta$ = the least k s.t. $\nu_i \leq \pi_{K_0 K_i}(\beta_k)$ for $i < \theta$, Define M, σ by: $M | \pi_{\kappa_0 \kappa_\theta}(\beta^k) = \kappa_\theta | \pi_{\kappa_0 \kappa_\theta}(\beta^k)$ and

$\sigma = \pi_{\kappa_0 \kappa_\theta} : M^k \underset{\Sigma_0}{\longrightarrow} M$ cofinally.

Then M is an iterate of M^k by the same indices. Moreover, $\tilde{\kappa} = (M^k)^\infty$, $\kappa_\theta = M^\infty$, and $\pi_{\kappa_0 \kappa_\theta} = \sigma^\infty$.

Hence it suffices to show:

<u>Claim</u> $M = N_\theta$.

We prove this by ind. on θ. Let it hold for $i < \theta$. Thus for $i < \theta$ we have:

(a) N_i is an iterate of M^{k_i} by the indices ν_h $(h < i)$.

(b) $\kappa_i = N_i^\infty$, $\pi_{\kappa_0 \kappa_i} = \sigma_i^\infty$, where

$\sigma_i = \pi_{M^{k_i}, N_i}$.

<u>Case 1</u> $\theta = 0$ immediate.

<u>Case 2</u> $\theta = i + 1$ and $\nu_i \leq \alpha_i$.

Then $k = k_i$ and N_θ is an iterate of N_i by the index ν_i, hence of M^k by the indices ν_h $(h \leq i)$. Hence $N_\theta = M$

<u>Case 3</u> $\theta = i+1$ and $\nu_i = \alpha_i + 1$

Then $k = k_i + 1$ and $K_\theta = K_i$.

$N_\theta = (N_i)^1$, since $\pi_{N_i N_\theta} : N_i \longrightarrow_{E_{\nu_i}} N_\theta$.

Hence $\pi_{K_0 K_\theta}(\beta^{k_i+1}) = \pi_{K_0 K_i}(\beta^{k_i+1}) =$

$= \alpha_\theta$, since $\pi_{K_0 K_i} = (\pi_{M^{k_i}, N_i})^\infty$.

Similarly $\pi_{K_0 K_\theta}(M^k \,|\, \beta^k) = N_\theta \,|\, \alpha_\theta$.

Hence $M \,|\, \alpha_\theta = N_\theta \,|\, \alpha_\theta$, where M, N_θ are s-mice. A comparison argument shows: $M = N_\theta$.

<u>Case 4</u> $\lim(\theta)$ and there is $i < \theta$ s.t. $k_j = k_i$ for $i \le j < \theta$.

Then $k = k_i$. N_θ is an iterate of N_i by the indices ν_j ($i \le j < \theta$), hence of M^k by ν_j ($j < \theta$).

Hence $N_\theta = M$.

<u>Case 5</u> The above cases fail.

Then $\lim(\theta)$ and $I = \{i \mid \nu_i = \alpha_i + 1\}$ is unbounded in θ

Then $k = \text{lub}\{k_i \mid i < \theta\}$. Set:
$\kappa = \sup\{\kappa_i \mid i \in I\}$. Then $\kappa = \pi_{N_i N_\theta}(\kappa_i)$
for $i \in I$ (where E_{ν_i} is on κ_i $(i \leq \theta)$),
Hence κ is the largest cardinal in N_θ.
Define τ_i $(i \in I)$ by: $\tau_i + \tilde{\kappa} = \beta^{k_i}$.
Then $\pi_{K_0 K_i}(\tau_i) = \pi_{K_0 K_\theta}(\tau_i) = \kappa_i$
for $i \in I$. Hence $\kappa = \sup\limits_i \tau_i =$
$= \pi_{K_0 K_\theta}(\tau_\theta)$, where $\tau_\theta = \sup\limits_i \tau_i$
(hence $\beta^k = \tau_\theta + \tilde{\kappa}$). But
$N_\theta | \kappa = \bigcup\limits_{i \in I} N_i | \kappa_i = \bigcup\limits_{i \in I} K_i | \kappa_i = K_\theta | \kappa$
κ is then the largest cardinal in
N_θ and M, where $N_\theta | \kappa = M | \kappa$.
Since both N_θ, M are 1-mice,
a comparison argument shows:
$M = N_\theta$. QED (Lemma 5)

The following definition enables us to state an obvious improvement of Lemma 5:

<u>Def</u> Let N_i $(i<\theta\le\infty)$ be an iteration of O^1 by indices $\langle \nu_i \mid \;_i \rangle$. We call K_i $(i<\theta)$ the iteration of $\tilde{K}$ by <u>corellated</u> <u>indices</u> iff it has indices $\langle \nu_i \mid \tilde{\;}_i \rangle$ where

$$\tilde{\;}_i = \begin{cases} On & \text{if } \omega_h = On \cap N_h \text{ for } h \le i \\ \;_i & \text{if not.} \end{cases}$$

<u>Cor 5.1</u> Let N_i $(i<\theta\le\infty)$ be a normal iteration of O^1 and let K_i $(i<\theta)$ be the iteration of $\tilde{K}$ by corellated indices. Then $N_i \mid \alpha_i = K_i \mid \alpha_i$ $(i<\theta)$ where $\alpha_i = On \cap N_i$

prf.
By Lemma 5 if the iteration is simple. Otherwise let i be least s.t. $\omega\alpha_i \in N_i$. It holds $\le i$ by Lemma 5 and hence $K_j = N_j$ for $j > i$. QED (5.1)

<u>Corollary 6</u> Let $\langle N_i \mid i < \infty \rangle$ be a normal iterate of O^{Λ}. Set $W = N_\infty = \bigcup_i J_{\kappa_i}^{E^{N_i}}$ (where E_{ν_i} is on ν_i if $E_{\nu_i} \neq \emptyset$). Then $W = K_\infty$ in the iterate of $\tilde{K}$ with corellated indices.

<u>Corollary 7</u> $\tilde{K}$ is universal.

pf.
Let M be a premouse which is coiterable with $\tilde{K}$. Coiterate M, O^{Λ} as far as possible, getting $\langle M_i \mid i < \theta \rangle$, $\langle N_i \mid i < \theta \rangle$ as iteration of M, O^{Λ} resp. $\theta < \infty$ by the usual argument. The coiteration either terminates in a comparable pair or cannot be continued because of ill foundedness. Let K_i $(i < \theta)$ be the iteration of $\tilde{K}$ by corellated indices. By Cor 5.1, $\langle K_i \rangle, \langle M_i \rangle$ is "essentially" the coiteration of $\tilde{K}, O^{\Lambda}$ up to θ (we need only omit points i s.t. $\nu_i = \alpha_i + 1$,

where $\alpha_i = \mathrm{On} \cap N_i$, since then neither K_i nor M_i is moved). Using the coiterability of $M, \tilde{K}$ it follows easily that the coiteration of M, O^2 would be continuable if it had not terminated. But this means that the coiteration of $M, \tilde{K}$ terminates by Cor 5.1.

QED (Cor 7)

We now prove a lemma which — in view of the weak covering lemma — shows how radically the universe is altered by the presence of O^2.

<u>Lemma 8</u> (Assume that O^2 exists) Let W be a weasel. Let β be a cardinal in W. Then $\mathrm{cf}(\beta^{+W}) = \omega$.

proof.
It suffices to prove the result for a simple iterate of W, since successors go cofinally to successors in ordinary iterations. Coiterate W, O^2. The iteration cannot terminate. Hence we

get W_i, N_i $(i < \infty)$ with:

$$W_\infty = \bigcup_i W | \nu_i = \bigcup_i N | \nu_i ,$$

where ν_i are the indices. W_∞ is a simple iterate of W by §3.1 Lemma 1.2

Hence it suffices to show:

<u>Claim</u> Let N be an iterate of $O^\#$. Let β be a cardinal in N. Then $\mathrm{cf}(\beta^+) = \omega$.

prf.

There is a countable iterate $\bar{N}$ of $O^\#$ s.t. N is an iterate of $\bar{N}$ and $\beta \in \mathrm{rng}(\pi_{\bar{N}N})$. Let $\pi_{\bar{N}N}(\bar{\beta}) = \beta$. Then $\bar{\beta}^{+\bar{N}}$ has cofinality ω, since $\bar{N}$ is countable. But $\pi_{\bar{N}N}$ takes successors cofinally to successors since we are working with ordinary ultraproducts. QED (Lemma 8)

We are now ready to prove:

Lemma 9 Let W be universal. Then W is a simple iterate of $\tilde{K}$.

proof.

By §3.2 it suffices to show that W is an iterate of $\tilde{K}$ or, in other words, that the coiteration of $\tilde{K}, W$ does not move W. But then it suffices to show that for each α, the coiteration of $\tilde{K}$, $W|\alpha$ does not move $W|\alpha$. If we form the canonical ω-complete weasel $\tilde{W} = W_\infty[W|\alpha]$ over $W|\alpha$, it suffices to show that the coiteration of $\tilde{K}, \tilde{W}$ does not move $\tilde{W}$. Hence it suffices to show:

Claim Let W be as in the covering lemma. Then the coiteration of $\tilde{K}, W$ does not move W.

The argument of Lemma 7 tells

us that if $\langle N_i \rangle, W$ is the coiteration of $O^\wedge, W$ (i.e. W is not moved) and K_i ($i < \infty$) is the iteration of $\tilde{K}$ by corellated indices, then $\langle K_i \rangle, W$ is "essentially" the coiteration of $\tilde{K}, W$ - i.e. W is not moved. Hence it suffices to show:

<u>Claim</u> The coiteration of $O^\wedge, W$ does not move W.

We prove this by reexamining the proof of the covering lemma. It is enough, of course, to show that the coiteration of $O^\wedge$ and an arbitrary $W|\gamma$ does not move $W|\gamma$. Choose a singular cardinal β s.t. $cf(\beta) \geq \gamma, \alpha_0^\omega$; where α_0 is as in the covering lemma. Since $\beta^{+W} < \beta^+$, the proof of the covering lemma at β must fail. Choose τ as before.

Then $\tau > \gamma$. Recall that we used $\neg O^{1}$ to prove Lemma 1, which said that there is a cub $C' \subset C$ s.t. $\mathbb{P}(\alpha) \cap W \not\subset W_\alpha$ for $\alpha \in C'$ s.t. $cf(\alpha) = \alpha_0$. We then derived a contradiction from Lemma 1, using only the universality of W. Thus Lemma 1 must fail and there is a stationary set $S \subset C$ s.t. $\mathbb{P}(\alpha) \cap W \subset W_\alpha$ and $cf(\alpha) = \alpha_0$ for $\alpha \in S$. As in Lemma 1 define ultrafilters U_α $(\alpha \in S)$ s.t. $\langle W|\alpha^+, U_\alpha \rangle$ is amenable ($\alpha^+ =_{Df} \alpha^{+W}$). As before, U_α is ω-complete on a stationary set $S' \subset S$. Pick $\alpha \in S'$ s.t. $\alpha \geq \gamma$. We obtained a contradiction by using $\neg O^{1}$ to conclude that $\langle W|\alpha^+, U_\alpha \rangle$ could not be an 1-mouse. Thus we must now conclude that $N = \langle W|\alpha^+, U_\alpha \rangle$ <u>is</u> an 1-mouse, hence an iterate of O^{1}. But then $W|\alpha^+$ was not moved in the coiteration of 0^1, $W|\alpha^+$. QED (Lemma 9)

<u>Cor 10</u> $\tilde{K} = K$

pf. Suppose not.

Then there is an α s.t. there is a strong mouse N of length α with $J_\alpha^{E^N} = J_\alpha^{E^{\tilde{K}}}$, $N \neq \tilde{K}|\alpha$, and $E_\alpha^N \neq \emptyset$. Let W be universal with $W|\alpha = N$. Coiterate $\tilde{K}$, W. Then W is moved. Contr! QED (Cor 10).

<u>Open Question</u> Is every weakly universal weasel an iterate of K? On the assumption $\neg 0^1$ we will show this to be the case.