Robust Extenders

In recent years there has been increased interest in weaker criteria of iterability. The traditional criterion was ω-completeness, which suffices for linear iterability. At the other end of the scale is the very strong criterion of "background certification", which Steel introduced in [S] and imposed on the extenders in the sequence used to define his K^c model. Because the criterion is hard to fulfill, most of his theorems must be stated relative to a universe V_Ω, where Ω is inaccessible or larger, rather than in ZFC. Hence there is interest in finding more modest criteria of iterability, which will work at least for restricted classes of premice. In [Sch] Ralf Schindler introduced the "almost linearly iterable" premice and showed that the criterion of ω-completeness sufficed for the construction of a K^c-like model for these structures. (In doing so, he actually rediscovered and modernized some ideas of Dodd, but he went further and constructed the core model for his structures in ZFC.) We extended some of his work in [TM]. Together with Qi Feng, we then tackled the class of premice ("type 1 premice") which comes just after the almost linearly iterable structures. This led to the notion of *supercompleteness*. In [FJ] we construct a K^c-type model in ZFC, using supercompleteness as our criterion of iterability. Feng has shown that this model is universal. Both the notions "ω-complete" and "supercomplete" are *self contained* in the sense, that they refer only to the structure (an extender in a premouse) and to countable sets of ordinals. Thus they are absolute in any inner model containing the structure and all countable sets of ordinals. The class of premice which Feng and I were able to handle was, however, very small by comparison with Steel's 1-small premice.

In the meantime Mitchell and Schindler had worked from the other end, weakening Steel's criterion so as to reduce his dependance on large cardinal universes. Their results are reported in [MSch], Mitchell can show their K^c-type model of 1-small mice to be universal under the assumptions:

ZFC + GCH + "There is no inner model with a Woodin cardinal".

In this paper we define a natural extension of supercompleteness which we call robustness. Like supercompleteness, the notion is self contained in the above sense. Nonetheless it appears to possess the efficacy of Steel's iteration criterion. Because it is self contained, we can build a K^c-type model for 1-small mice and remove the hypothesis GCH from Mitchell's universality result.

In this paper we work with "λ-pre-mice" as developed in [NFS] and [CR]. We think it very likely that the same constructions can be carried out for premice with other indexings.

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