Appendix to "Subproper and Subcomplete Forcing"

Since writing this paper we have made some revisions in §0 and §2. This enabled us to correct some slips of the pen and improve the exposition at a couple of places. The main purpose of the revision, however, was to correct a conceptual error: In order that $\sigma : \bar{N} \prec N$ witness the subproperness of $\mathbb{B}$, we required that a certain condition be satisfied for arbitrarily chosen $\lambda_1, \ldots, \lambda_m \in \mathrm{rng}(\sigma)$ s.t. $\mathbb{B} \in H_{\lambda_i}$ and λ_i is regular for $i = 1, \ldots, n$. Taken literally, this would mean that subproperness is not a structural property of $\mathbb{B}$ - i.e. it could hold for $\mathbb{B}$ without holding for all structures isomorphic to $\mathbb{B}$. (Indeed, subproperness gets easier to satisfy as the rank of $\mathbb{B}$ increases, since there are then fewer λ with $\mathbb{B} \in H_\lambda$.) As is to be expected, this carelessness caused us to state things which are not literally true. Thus e.g. in the proof of the iteration theorem for subproperness (Theorem 5) we stated that in Case 3.2 we have $\mathbb{B}_i \in H_\lambda$ for $i < \lambda$. In fact, all we know is that $\overline{\overline{\mathbb{B}_i}} < \lambda$. Thus we have changed the definition of subproperness in §1 to require the abovementioned condition to hold for arbitrary regular $\lambda_1,\dots,\lambda_n \in \mathrm{rng}(\sigma)$ s.t. $\overline{\overline{\mathbb{B}}} < \lambda_i$ $(i=1,\dots,n)$. With this change the proof of the iteration theorems go through. It means, however, that it has become marginally harder to verify the subproperness of a given $\mathbb{B}$. In §3 we verify subproperness and subcompleteness for various sets of conditions $\mathbb{P}$. (Let us call $\mathbb{P}$ subproper if $BA(\mathbb{P})$ is a subproper Boolean algebra.) In all of these verifications, we verify the abovementioned condition for an arbitrarily chosen sequence $\lambda_1,\dots,\lambda_n \in \mathrm{rng}(\sigma)$ s.t. λ_i is regular and $\mathbb{P} \in H_{\lambda_i}$ for $i=1,\dots,n$. This is harmless, however, if $\mathbb{P}$ is so chosen that $\overline{\overline{\mathbb{P}}} = \overline{\overline{TC(\mathbb{P})}}$ ($TC(\mathbb{P})$ being the transitive closure of $\mathbb{P}$). We then have $\mathbb{P} \in H_\lambda$ iff $\overline{\overline{\mathbb{P}}} < \lambda$. We have then certainly proven enough, since $\overline{\overline{BA(\mathbb{P})}} < \lambda$ implies $\overline{\overline{\mathbb{P}}} < \lambda$. In fact, all of the sets of conditions considered in §3 have the property $\overline{\overline{\mathbb{P}}} = \overline{\overline{TC(\mathbb{P})}}$, so no change to §3 is required.

This brings us, however, to another discrepancy in our treatment. We just defined $\mathbb{P}$ to be subproper if $BA(\mathbb{P})$ is subproper. This is, however, not the definition given in §0, where we wrote $\overline{\overline{\mathbb{P}}} < \lambda_i$ rather than $\overline{\overline{BA(\mathbb{P})}} < \lambda_i$. Thus subproperness in the sense of §0 is harder to satisfy. This suggests a strengthened notion of subproperness for complete BA's $\mathbb{B}$: Set $d(\mathbb{B})$ = the smallest cardinality of a set dense in $\mathbb{B} \setminus \{0\}$. We then obtain the notion of strong subproperness by writing "$d(\mathbb{B}) < \lambda_i$" instead of "$\overline{\overline{\mathbb{B}}} < \lambda_i$". For this notion we obtain the following iteration lemma:

Let $\mathbb{B} = \langle \mathbb{B}_i \mid i \leq \alpha \rangle$ be an RCS iteration s.t. $\Vdash_i ((\check{\mathbb{B}}_{i+1} / \dot{G}_i)$ is strongly subproper$)$ for $i < \alpha$. Suppose, moreover, that $\xi \leq d(\mathbb{B}_\xi)$ and $\mathbb{B}_{\xi+1}$ collapses $d(\mathbb{B}_\xi)$ to ω_1 for $\xi < \alpha$. Then each $\mathbb{B}_\xi$ is strongly subproper.

The same proof works.