AN APPROXIMATION METHOD IN STOCHASTIC OPTIMAL CONTROL

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1. Introduction

In the following we will consider stochastic optimal control problems in which the dynamical system is described by a nonlinear random ordinary differential equation and the functional is established with respect to the expected value:

$$\inf_{u \in C} J(u) \qquad \text{(P)}$$

where $J(u) := E\{g(x^1, x(T))\}$ (1)

$\dot{x}(t) = f(t, z(t), u(t), x(t))$, $t \in [t_0, T]$, $x(t_0) = x^0$ (2)

C is a nonempty set of deterministic or stochastic controls. (3)

x^0, x^1 are random variables and z is a stochastic process defined on a probability space $(\Omega, \mathfrak{A}, P)$. (2) represents an initial value problem for a random ordinary differential equation and E is the symbol for the expected value. Possible integral-parts in the functional (1) let be transformed by introducing new state variables.

The following investigation aims at the application of a Ritz-Galerkin method for the approximate solving of (P). This method is based on an approximation of stochastic processes by processes with finitely many realizations (see chapt. 2). Besides a general convergence theorem we obtain in chapter 3 that the approximate problems are completely deterministic ones. In the case of deterministic controls each of these problems represents a certain deterministic optimal control problem and in the case of stochastic controls a family of deterministic optimal control problems.

We investigate (P) under the following general supposition (S):

(i) $g: R^n \times R^n \longrightarrow R^1$ is continuous and there exist constants $L > 0$, $a > 0$, $b \in R^1$, $p \in [1, \infty)$ such that $|g(y_1, y_2)| \leq a + L(|y_1|^p_{R^n} + |y_2|^p_{R^n})$ and $g(y_1, y_2) \geq b$ holds for all $y_1, y_2 \in R^n$.

(ii) $f: [t_0, T] \times R^s \times R^r \times R^n \longrightarrow R^n$ is continuous and for all $z_i \in R^s, u_i \in R^r$, $x_i \in R^n$, $i=1,2$, $t \in [t_0, T]$
$|f(t, z_1, u_1, x_1) - f(t, z_2, u_2, x_2)|_{R^n} \leq L(|z_1 - z_2|_{R^s} + |u_1 - u_2|_{R^r} + |x_1 - x_2|_{R^n})$
holds.

(iii) $(\Omega, \mathfrak{A}, P)$ is a probability space, $x^0, x^1 \in L^n_p(\Omega, \mathfrak{A}, P)$,

$z \in L_1([t_o,T],L_p^s(\Omega,\mathfrak{A},P))$.

(iv) $\emptyset \neq C \subset L_1([t_o,T],L_p^r(\Omega,\mathfrak{A},P))$.

Remark 1:

a) Under the supposition (S) the random differential equation (2) can be understood as an abstract differential equation in the Banach space $B=L_p^n(\Omega,\mathfrak{A},P)$. The operator $f:[t_o,T]\times L_p^s(\Omega,\mathfrak{A},P)\times L_p^r(\Omega,\mathfrak{A},P)\times L_p^n(\Omega,\mathfrak{A},P)\longrightarrow L_p^n(\Omega,\mathfrak{A},P)$ then satisfies a Lipschitz condition that is analogous to (ii). Hence an unique solution $x\in C([t_o,T],L_p^n(\Omega,\mathfrak{A},P))$ exists for each $u\in L_1([t_o,T],L_p^r(\Omega,\mathfrak{A},P))$ (comp. [11], p.541; [5]; [9], Theorem 1).

b) If $x,\tilde{x}$ are the -according to a) uniquely determined- solutions of the random differential equation (2) with the input parameters x^o, z,u and $\tilde{x}^o,\tilde{z},\tilde{u}$, respectively, then (S) and Gronwall's inequality (comp. [12], p.189) yield the estimation:

$$\|x(t)-\tilde{x}(t)\|_{L_p^n} \leq e^{L(t-t_o)}\Big(\|x^o-\tilde{x}^o\|_{L_p^n} + L\int_{t_o}^{t} \big(\|z(s)-\tilde{z}(s)\|_{L_p^s} + \|u(s)-\tilde{u}(s)\|_{L_p^r}\big)ds \Big) \quad \text{, for each } t\in[t_o,T].$$

c) According to (S)(i) g can be considered to be a continuous operator $g: L_p^n(\Omega,\mathfrak{A},P)\times L_p^n(\Omega,\mathfrak{A},P)\longrightarrow L_1(\Omega,\mathfrak{A},P)$ and the functional (1) is uniquely defined for each $u\in L_1([t_o,T],L_p^r(\Omega,\mathfrak{A},P))$ (resp. for $u\in L_1([t_o,T],R^r)$ especially). Furthermore the condition (S)(i) secures $\inf_{u\in C} J(u) > -\infty$. It turns out that (P) can be formulated as a minimum problem in the Banach space $L_1([t_o,T],L_p^r(\Omega,\mathfrak{A},P))$.

2. An approximation of stochastic processes

In the following let $(\Omega,\mathfrak{A},P)$ be a probability space, $(R^n,\mathfrak{B}^n)$ the Borel measurable space and $I\subseteq R^1$. Let $x: I\times\Omega\longrightarrow R^n$ denote a vector stochastic process with the state space R^n and the parameter set I, $\mathfrak{A}_x$ be the smallest σ-algebra with respect to which each random variable $x(t)$, $t\in I$, of the process is measurable. For a random variable $z:\Omega\longrightarrow R^n$ defined on $(\Omega,\mathfrak{A},P)$ let $E(z|A)$ $(A\in\mathfrak{A}, P(A)>0)$ denote the conditional expected value of z relative to the event A. $L_p^n(\Omega,\mathfrak{A},P)$ with the norm $\|z\|_p := [E(|z|_{R^n}^p)]^{\frac{1}{p}}$ shall denote the Banach space of random variables defined on $(\Omega,\mathfrak{A},P)$ being integrable to the p(th) power $(1\leq p<\infty)$. Moreover, let a sequence $\{\{A_l^m\}_{l=1,\dots,s_m}\}_{m\in N}$ be given with the following properties:

(i) $\forall m \in N: \{A_l^m\}_{l=1,\dots,s_m}$ is a finite decomposition of Ω ,

(ii) $\forall m \in N: \mathfrak{A}_m \subseteq \mathfrak{A}_{m+1}$, where $\mathfrak{A}_m := \mathfrak{A}(\{A_l^m\}_{l=1,\dots,s_m})$,

(iii) $\mathfrak{A}_x \underset{=P}{\subseteq} \mathfrak{A}(\bigcup_{m \in N} \mathfrak{A}_m)$

(Here $\mathfrak{A}(\mathcal{E})$ is the smallest σ-algebra that contains $\mathcal{E} \subseteq \mathfrak{A}$ and it holds that $\bar{\mathfrak{A}} \underset{=P}{\subseteq} \tilde{\mathfrak{A}}$ if for each $\bar{A} \in \bar{\mathfrak{A}}$ there exists an event $\tilde{A} \in \tilde{\mathfrak{A}}$ with $P(\bar{A} \Delta \tilde{A})=0$.)

If we now suppose that $x: I \longrightarrow L_1^n(\Omega, \mathfrak{A}, P)$ and if we define for each $m \in N$ the following processes with finitely many realizations

$$x_m(t) := \sum_{l=1}^{s_m} E(x(t)|A_l^m)\, 1_{A_l^m} \quad , t \in I,$$

then the following convergence statement is valid (see[8], Theorem 4):

<u>Theorem 1</u>:

Suppose that $x:I \longrightarrow L_1^n(\Omega, \mathfrak{A}, P)$ and $x_m, m \in N$, are defined as above, then:

a) For all $t \in I$ it holds that:

$$\lim_{m \to \infty} |x_m(t,\omega) - x(t,\omega)|_{R^n} = 0 \quad \text{a.s.}$$

$$\lim_{m \to \infty} \|x_m(t) - x(t)\|_p = 0 \quad , \text{ if } x(t) \in L_p^n(\Omega, \mathfrak{A}, P) \ , \ 1 \le p < \infty .$$

b) If I is measurable, bounded and $x \in L_q(I, L_p^n(\Omega, \mathfrak{A}, P))(1 \le p,q < \infty)$, then it holds that:

$$\lim_{m \to \infty} \int_I \|x_m(t) - x(t)\|_p^q \, dt = 0$$

c) If I is a compact interval and $x \in C(I, L_2^n(\Omega, \mathfrak{A}, P))$, then

$$\lim_{m \to \infty} \max_{t \in I} \|x_m(t) - x(t)\|_2 = 0 .$$

The proof is a consequence of continuity properties of the conditional expectation.

<u>Remark 2</u>:

a) Under some weak suppositions to x (comp.[7],[8]) there exist such sequences $\{\{A_l^m\}_{l=1,\dots,s_m}\}_{m \in N}$ of finite decompositions of Ω.
According to the way in which they are generated there exist possibilities for the computation of the realizations $E(x(.)|A_l^m)$ of the approximate process x_m and their probabilities $P(A_l^m)$, $l=1,\dots,s_m$.

b) Provided that there exists a sequence $\{z_i\}_{i \in N}$ of real random variables defined on $(\Omega, \mathfrak{A}, P)$ with $(*)$ $\mathfrak{A}_x \underset{=P}{\subseteq} \mathfrak{A}(\bigcup_{i \in N} \mathfrak{A}_{z_i})$, then such sequences as in a) can be generated - by means of families of finite decompositions $\{I_{j,i}^n\}_{j=1,\dots,n} \subset \mathfrak{B}^1$, $i,n \in N$, of R^1 under further proper suppositions - as follows:

$$A_l^{(n,k)} := \bigcap_{i=1}^{k} z_i^{-1}(I^n_{l_i,i}) \quad , \; l=1,\dots,n^k, \quad l=1+\sum_{i=1}^{k}(l_i-1)n^{i-1} \; ,$$
$$l_i \in \{1,\dots,n\},\; n,k \in N.$$

Obviously $P(A_l^{(n,k)})$ and $E(x(t)|A_l^{(n,k)})$, respectively, are determined by the distribution of $(z_1,\dots,z_k)^T$ and $(x(t),z_1,\dots,z_k)^T$ ([8]).

c) For the case of a real Gaussian process an effective algorithm for the computation of the realizations $E(x(.)|A_l^m)$ was given in [8], chapter 6. There the $\{z_i\}_{i\in N}$ are chosen as proper independent $N(0,1)$-distributed real random variables. Convergence $\lim_{m\to\infty} \|x_m(t) - x(t)\|_2 = 0$ for each $t \in I$ is already secured for processes being continuous in probability. For the carrying out of the algorithm we only need the mean- and the covariance function of the process.

d) The represented approximation of stochastic processes is in its kind related to Monte-Carlo methods. Contrary to the Monte-Carlo methods (pseudo-) random number generators are not needed.

3. A Ritz-Galerkin method in stochastic optimal control

For the approximation of stochastic optimal control problems (P) we now use a Ritz-Galerkin method. This method is based on the represented approximation of stochastic processes. The approximate problems result by replacing the stochastic input parameters x^o, x^1, z by the corresponding approximations x^o_m, x^1_m, z_m and by minimizing in proper subsets C_m of C. The approximate problems consequently are of the form

$$\inf_{u_m \in C_m} J_m(u_m) \tag{Pm}$$

where $J_m(u_m) := E\{g(x^1_m, x_m(T))\}$ (1m)

$$\dot{x}_m(t) = f(t, z_m(t), u_m(t), x_m(t)) \;, t\in[t_o,T] \;, x_m(t_o)=x^o_m \tag{2m}$$

$$\emptyset \neq C_m \subset L_1([t_o,T], L^r_p(\Omega, \alpha_m, P)) \; . \tag{3m}$$

The problem (Pm) represents a Ritz-Galerkin approximation to (P). Since the usual simple convergence proofs for ordinary Ritz methods do not apply (see [3]), we use a general approximation scheme according to [1]. In the following we denote by $[C,J]$ the minimum problem (P) and by $[C_m,J_m]$ the problem (Pm). As in [1] we define:

$\{[C_m,J_m]\}_{m\in N}$ approximates $[C,J]$ iff $\lim_{m\to\infty} J^*_m = J^* := \inf_{u\in C} J(u)$,

where $J^*_m := \inf_{u_m\in C_m} J_m(u_m)$.

Lemma:

$\{[C_m,J_m]\}_{m\in N}$ approximates $[C,J]$ iff

(I) to each $u\in C$ and each $m\in N$ there exists a $v_m\in C_m$ with $\overline{\lim}_{m\to\infty} J_m(v_m) \leq J(u)$.

(II) for each sequence $\{v_m\}_{m\in N}$, $v_m\in C_m$, with $\lim_{m\to\infty}(J_m(v_m)-J_m^*)=0$, there exists a sequence $\{u_m\}_{m\in N}\subset C$ such that $\overline{\lim}_{m\to\infty}(J(u_m)-J_m(v_m))\leq 0$.

Proof: [1], p.157 .

Theorem 2:

Supp.: a) Let (S) be fulfilled; the stochastic process $(x^o,x^1,z(.))^T$ defined on $(\Omega,\mathfrak{A},P)$ with the state space R^{2n+s} satisfies the condition ($*$) in remark 2b); $\mathfrak{A}_m := \mathfrak{A}(\{A_l^m\}_{l=1,\dots,s_m})$, x_m^o, x_m^1, z_m, $m\in N$, are defined as in chapter 2.

b) $C\subset L_1([t_o,T],L_p^r(\Omega,\mathfrak{A},P))$ and $\mathfrak{A}_m \subseteq_P \bar{\mathfrak{A}}$, where $\bar{\mathfrak{A}} := \mathfrak{A}(\mathfrak{A}_{x^o}\cup\mathfrak{A}_{x^1}\cup\mathfrak{A}_z)$.

c) If $Q_m: L_1([t_o,T],L_p^r(\Omega,\mathfrak{A},P)) \longrightarrow L_1([t_o,T],L_p^r(\Omega,\mathfrak{A}_m,P))$ denotes the operator $(Q_m u)(t) := \sum_{l=1}^{s_m} E(u(t)|A_l^m)\, 1_{A_l^m}$, $t\in[t_o,T]$, then $Q_m(C)\subseteq C_m$ holds for $m\in N$.

Then $\{[C_m,J_m]\}_{m\in N}$ approximates $[C,J]$.

Proof:

a) We aim at applying the lemma and start with proving condition (I) by showing that $\lim_{m\to\infty} J_m(Q_m u) = J(u)$ for each $u\in C$.

For $u\in C$ let x and x_m, respectively, denote the solution of (2) and (2m) with $u_m := Q_m u$. Then we obtain

$$|J(u) - J_m(Q_m u)| \leq E\{|g(x^1,x(T)) - g(x_m^1,x_m(T))|\}$$

and according to remark 1b)

$$\|x(T)-x_m(T)\|_{L_p^n} \leq e^{L(T-t_o)}(\|x^o-x_m^o\|_{L_p^n} + L\int_{t_o}^{T}(\|z(s)-z_m(s)\|_{L_p^s} + \|u(s)-(Q_m u)(s)\|_{L_p^r})\,ds) .$$

This estimation, the definition of Q_m, the supposition and Theorem 1 prove $\lim_{m\to\infty}\|x(T) - x_m(T)\|_{L_p^n} = 0$.

The continuity of g completes the proof of condition (I).

b) In order to prove condition (II) we have to show that
$\lim_{m\to\infty} |J(v_m) - J_m(v_m)| = 0$ for arbitrary sequences $\{v_m\}_m$, $v_m \in C_m$, $m\in N$.
Now let $\tilde{x}_m$ and x_m, respectively, denote the solution of (2) with $u:=v_m$ and of (2m) with $u_m:=v_m$. Again using remark 1b) we obtain

$$\|\tilde{x}_m(T) - x_m(T)\|_{L_p^n} \leq e^{L(T-t_o)}(\|x^o - x_m^o\|_{L_p^n} + L\int_{t_o}^{T} \|z(s)-z_m(s)\|_{L_p^s}\, ds)$$

and therefore

$$\lim_{m\to\infty} |J(v_m) - J_m(v_m)| \leq E\{|g(x^1,x_m(T)) - g(x_m^1,x_m(T))|\} \xrightarrow[m\to\infty]{} 0 .$$

q.e.d.

<u>Remark 3</u>:

a) The case of deterministic controls:
We suppose that $\emptyset \neq C \subset L_1([t_o,T],R^r)$, $C_m := C$, $m\in N$. In this case the suppositions b) and c) of Theorem 2 are trivially fulfilled. According to chapter 2 x_m^o, x_m^1, z_m have representations of the form:

$$x_m^o = \sum_{l=1}^{s_m} x_{m,l}^o \, 1_{A_l^m} \quad , \quad x_{m,l}^o := E(x^o | A_l^m) \in R^n ,$$

$$x_m^1 = \sum_{l=1}^{s_m} x_{m,l}^1 \, 1_{A_l^m} \quad , \quad x_{m,l}^1 := E(x^1 | A_l^m) \in R^n ,$$

$$z_m(\cdot) = \sum_{l=1}^{s_m} z_{m,l}(\cdot) \, 1_{A_l^m} \quad , \quad z_{m,l}(\cdot) := E(z(\cdot) | A_l^m) \in L_1([t_o,T],R^s) .$$

Then the solution x_m of (2m) has the representation (comp. e.g.[8])

$x_m(\cdot) = \sum_{l=1}^{s_m} x_{m,l}(\cdot)\, 1_{A_l^m}$, where $x_{m,l} \in C([t_o,T],R^n)$ is the solution of the deterministic ordinary differential equation

$\dot{x}_{m,l}(t) = f(t,z_{m,l}(t),u(t),x_{m,l}(t))$, $t\in[t_o,T]$, $x_{m,l}(t_o) = x_{m,l}^o$, $l=1,\dots,s_m$. Moreover the following representation for (1m) results:

$$J_m(u) = \sum_{l=1}^{s_m} g(x_{m,l}^1, x_{m,l}(T))\, P(A_l^m) \quad , \quad u\in C .$$

Thus, if $\{x_{m,l}^o, x_{m,l}^1, z_{m,l}, P(A_l^m)\}_{l=1,\dots,s_m}$ are known, then the approximate problem (Pm) represents a completely deterministic optimal control problem. With respect to the deterministic control problem corresponding to (P) (with deterministic input parameters x^o, x^1, z) only the outer form of the functional and the dimension of the system of the o.d.e. changed. The properties of the problem (1m), which e.g. make a numerical treatment possible, remained unchanged.

b) The case of stochastic controls:

We suppose $\emptyset \neq C := \{ u \in L_1([t_o,T],L_p^r(\Omega,\bar{\alpha},P)) | u(.,\omega) \in C_d \text{ a.s.}\}$, $C_d \subset L_1([t_o,T],R^r)$ and $C_m := C \cap L_1([t_o,T],L_p^r(\Omega,\alpha_m,P))$.
If x_m^o, x_m^1, z_m are of the form as in a) and if $u_m \in C_m$ is of the form $u_m(.) = \sum_{l=1}^{s_m} u_{m,l}(.)\, 1_{A_l^m}$, $u_{m,l} \in C_d$ according to the supposition, then the representation

$$J_m(u_m) = \sum_{l=1}^{s_m} g(x_{m,l}^1, x_{m,l}(T))\, P(A_l^m)$$

$$\dot{x}_{m,l}(t) = f(t,z_{m,l}(t),u_{m,l}(t),x_{m,l}(t)) \;,\; t\in[t_o,T],\; x_{m,l}(t_o)=x_{m,l}^o$$

results for (1m) and (2m), respectively.
In this case the problem (Pm) is equivalent to the following family of deterministic optimal control problems:

$$\{ \inf_{u_{m,l} \in C_d} J_{m,l}(u_{m,l}) \}_{l=1,\ldots,s_m}$$

where $J_{m,l}(u_{m,l}) := g(x_{m,l}^1, x_{m,l}(T))$

$$\dot{x}_{m,l}(t) = f(t,z_{m,l}(t),u_{m,l}(t),x_{m,l}(t)) \;,\; t\in[t_o,T],\; x_{m,l}(t_o)=x_{m,l}^o$$

$(l=1,\ldots,s_m)$

Each of these deterministic problems represents just a deterministic optimal control problem corresponding to (P).

Remark 4:

a) If e.g. C_d is of the form $C_d := \{u \in L_1([t_o,T],R^r) | u(t) \in D(t) \text{ a.e.}\}$ with convex, closed $D(t)$, $t\in[t_o,T]$, then $E(u(.)|A_l^m) \in C_d$, $l=1,\ldots,s_m$, results from $u(.,\omega) \in C_d$ a.s. (see [12],p.145) and supposition c) of Theorem 2 is fulfilled.

b) The condition $\alpha_m \subseteq_P \bar{\alpha}$ is fulfilled in most cases ([8]). In some special cases we can do without the condition $C \subset L_1([t_o,T],L_p^r(\Omega,\bar{\alpha},P))$ (see e.g.[9]).

c) If we succeed in determining a sequence $u_m^* \in C_m$, $m \in N$, in such a way that $J_m^* \leq J_m(u_m^*) \leq J_m^* + \varepsilon_m$, $\{\varepsilon_m\}_{m\in N}$ a positive zero sequence, then $\{u_m^*\}_{m\in N}$ is a minimizing sequence for J on C. Statistical characteristics of u_m^*, $m\in N$, such as the expected value, moments and distribution can be computed in a very simple way ([8]).
Hence the task of practically realizing the approximation method

consists in the approximate solution of the deterministic optimal control problems being equivalent to (Pm).

Remark 5:

a) If the functional J in (P) has more generally the form

$$J(u) := E\left\{\int_{t_0}^{T} f_0(t,z(t),u(t),x(t))dt + g(x^1,x(T))\right\} \quad , \qquad (1)'$$

then this stochastic control problem can, as usual (comp. e.g.[4], p.41), be transformed into a problem of the type (P). For this purpose we introduce a new state variable x_0 and the differential equation $\dot{x}_0(t)=f_0(t,z(t),u(t),x(t))$, $t\in[t_0,T]$, $x_0(t_0)=0$, and we add it to the state equations. With the notations $\hat{x}:=(x_0,x)^T$, $\hat{f}:=(f_0,f)^T$, $\hat{x}^0:=(0,x^0)^T$ the following problem of the type (P) results:

$$J(u) := E\{x_0(T) + g(x^1,x(T))\}$$

$$\dot{\hat{x}}(t) = \hat{f}(t,z(t),u(t),\hat{x}(t)) \ , \ t\in[t_0,T], \ \hat{x}(t_0) = \hat{x}^0 \ .$$

This remark aims at pointing out that the approximation method presented in this chapter can be applied to the more general problem even if $\hat{f}$ does *not* satisfy an assumption like (S)(ii). For example, it is sufficient that f satisfies (S)(ii) and that the following supposition is fulfilled for f_0:

$$f_0: [t_0,T]xR^sxR^rxR^n \longrightarrow R^1 \qquad f_0(t,z,u,x):=f_{01}(t,z.x)+f_{02}(t,u) \ ,$$

where $f_{01}: [t_0,T]xR^sxR^n \longrightarrow R^1$ and $f_{02}: [t_0,T]xR^r \longrightarrow R^1$ are continuous and the conditions

$$|f_{01}(t,z,x)| \leq c(t)+L(|z|^p_{R^s} + |x|^p_{R^n}) \quad , \quad |f_{02}(t,u)| \leq c(t)+L|u|^p_{R^r}$$

are satisfied for each $t\in[t_0,T], z\in R^s, u\in R^r, x\in R^n$ and for suitable $c\in L_1([t_0,T])$ and $L>0$.

Under these conditions the mappings

$$f_{01}(t,.,.): L^s_p(\Omega,\alpha,P)xL^n_p(\Omega,\alpha,P) \longrightarrow L_1(\Omega,\alpha,P) \quad \text{and}$$

$$f_{02}(t,.): L^r_p(\Omega,\alpha,P) \longrightarrow L_1(\Omega,\alpha,P) \quad \text{are continuous for all t.}$$

Having a look at the proof of Theorem 2 and at the remark 1b) shows that, due to this, the convergence statements remains valid for the general problem, too.

A special case of (P) with a functional (1)' is the Tracking-problem with a quadratic functional and a nonlinear random differential equation.

b) An extension of the results seems possible, e.g. for weakened suppositions to f and g, for problems with state and stochastic constraints.

4. Conclusions

An essential intention of the carried out investigations consisted in showing the applicability of the approximation principle for stochastic processes (see chapter 2) for the approximation of stochastic optimal control problems. After results for random ordinary differential equations ([7],[8]) and the Tracking-problem with linear random control equation and linear control constraints ([9]) had already been available, the results were to be expanded on more general nonlinear optimal control problems. For this and for the numerical, computational realization of the approximation methods further investigations are necessary. First numerical results for the already mentioned case of the Tracking-problem with a random control equation and stochastic controls we obtained in [9]. For the general method an algorithm was developed there and an ALGOL-programme was worked out for the case of Gaussian stochastic input parameters x^0,z. The occuring deterministic optimal control problems (comp. remark 3) were approximately solved by a method of conditional gradients and approximations of various statistical characteristics of the optimal control were computed. The results showed the applicability of the suggested methods. This fact, we call it the universality of the approximation method, seems to be an essential advantage. A disadvantage is the high expense of the method when using a great number s_m of realizations. That is why a proper a priori selection of m and s_m, respectively, is of great importance (comp.[8], chapter 6). But in general we have to make the best of this disadvantage since there seem to be no efficient methods of another type which can be used for the approximate solution of general nonlinear stochastic control problems.

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