Lecture notes SFT VIII

Precourse

Symplectic homology

Alexander Fauck

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Good overview articles : [4] and [5].

1 Floer homology on non-compact manifolds

- Symplectic homology is an attempt to generalize Floer homology to non-compact symplectic manifolds
- Let (V, ω) be a symplectic manifold, J an ω-compatible almost complex structure and $H : S^1 \times V \to \mathbb{R}$ a Hamiltonian
- What could go wrong if we want to define $FH(H)$ via the Floer equation

$$\partial_s u + J(u)\big(\partial_t u - X_H(u)\big) = 0 \quad ? \tag{$*$}$$

  - $FC_*(H)$ could be of infinite rank even for fixed degree and then $\partial^F x$ might involve infinite sums $\Rightarrow$ could be not well-defined
  - For fixed 1-periodic orbits x, y of X_H the solutions u of $(*)$ with $\lim_{s\to-\infty} u = x$ and $\lim_{s\to+\infty} u = y$ might not be contained in a compact subset of V $\Rightarrow \mathcal{M}(x, y)$ has no nice compactification.

$\Rightarrow$ We have to restrict the class of open symplectic manifolds and the class of Hamiltonians

- Let (V, ω) be a compact symplectic manifold with positive contact type boundary $\Sigma = \partial V$, i.e. near Σ, there exists a vector field Y such that $\mathcal{L}_Y\omega = \omega$ and Y points out of V along Σ. By the way, Y is called a Liouville field for ω.

- Define the Liouville form λ by $\lambda := \iota_Y \omega$ and note that $\mathcal{L}_Y \lambda = \lambda$. It restricts to a contact form $\alpha := \lambda|_{T\Sigma}$ on Σ with Reeb vector field $\mathcal{R}$ and contact structure $\xi := \ker \alpha$.
- The flow φ^t of Y for $t \in (-\varepsilon, 0]$ symplectically identifies a collar neighbourhood of Σ with $\big(\Sigma \times (-\varepsilon, 0], d(e^r \cdot \alpha)\big)$. Define the *completion* $(\widehat{V}, \widehat{\omega})$ by

$$\widehat{V} = V \cup_{\varphi^t} \Sigma \times (-\varepsilon, \infty)$$
$$\widehat{\omega} = \begin{cases} \omega & \text{on } V \\ d(e^r \cdot \alpha) & \text{on } \Sigma \times (-\varepsilon, \infty) \end{cases}.$$

- Call H cylindrical at infinity if there exists $R > -\varepsilon$ and a function $h : \mathbb{R} \to \mathbb{R}$ such that

$$H(y, r) = h(e^r) \qquad \text{on} \qquad \Sigma \times [R, \infty) \subset \widehat{V}$$

Note that on $\Sigma \times [R, \infty)$ we have $X_H = h'(e^r) \cdot \mathcal{R}$, as $dH_{(y,r)} = h'(e^r) \cdot d(e^r)$ and $\omega_{(y,r)} = d(e^r) \wedge \alpha + e^r \cdot d\alpha$.
- Call J cylindrical at infinity if it is time independent and of the form

$$J\xi = \xi \text{ and } J\partial_r = \mathcal{R} \qquad \Leftrightarrow \qquad d(e^r) \circ J = -\lambda \qquad \text{on } \Sigma \times [R, \infty).$$

Proposition (Maximum Principle)**.** *Let H_s, J be cylindrical at infinity with $\partial_s h'_s \leq 0$ and let $u : \mathbb{R} \times S^1 \to \widehat{V}$ be a solution of $(*)$ such that $\lim_{s \to \pm\infty} u \subset \widehat{V} \setminus \Sigma \times [R, \infty)$. Then*

$$u(\mathbb{R} \times S^1) \subset \widehat{V} \setminus \Sigma \times [R, \infty).$$

Remark. A similar result holds, if H_s is of the form $H_s(y, r) = h_s(e^{r - f_s(y)})$, i.e. if we consider a different contact form $\alpha' := e^{f(y)} \cdot \alpha$. The proof is similar but more involved (see [2]).

Lemma (E. Hopf's Weak Maximum Principle)**.** *Let $\Omega \subset \mathbb{R}^n$ be a bounded open domain. Consider a differential operator of the form*

$$L := \sum_{k=1}^{n} \frac{\partial^2}{\partial^2 x_k} + \sum_{k=1}^{n} b_k(x) \frac{\partial}{\partial x_k}$$

such that the functions b_k are uniformly bounded on Ω. If ρ is a C^2-function on $\overline{\Omega}$ such that $L\rho \geq 0$, then ρ attains its maximum on $\partial\Omega$.

Proof:

1. Assume that $L\rho > 0$. As ρ is continuous, it attains its maximum on $\overline{\Omega}$. If ρ attains the maximum at $x_0 \in \Omega$ then

$$\frac{\partial \rho}{\partial x_k}(x_0) = 0 \qquad \text{and} \qquad \frac{\partial^2 \rho}{\partial^2 x_k}(x_0) \leq 0 \quad \forall k.$$

   Hence $L\rho \leq 0$, a contradiction.

2. Now assume the general case $L\rho \geq 0$. Assume that $\overline{\Omega}$ is inside $\{|x_1| \leq d\}$. Consider $\sigma(x) := \rho(x) + \varepsilon \cdot e^{\alpha \cdot x_1}$ with $\alpha, \varepsilon > 0$. Then

$$\begin{aligned} L\sigma &= L\rho + \varepsilon \cdot \big(\alpha^2 + \alpha b_1(x)\big) e^{\alpha \cdot x_1} \\ &\geq \varepsilon \cdot \big(\alpha^2 - \alpha ||b_1||_\infty\big) e^{\alpha \cdot x_1}. \end{aligned}$$

$\Rightarrow$ for α large enough, we find $L\sigma > 0$. By 1. σ attains its maximum at the boundary

$$\Rightarrow \quad \sup_\Omega \rho \leq \sup_\Omega \sigma = \sup_{\partial\Omega} \sigma \leq \sup_{\partial\Omega} \rho + \varepsilon \cdot e^{\alpha d}$$

With $\varepsilon \to 0$ we find the Lemma.

□

Proof of the Proposition. We consider the function $\rho = e^r \circ u$. Then

$$\begin{aligned} \partial_s \rho = d(e^r)(\partial_s u) &= d(e^r)\big(-J(\partial_t u - X_H)\big) \\ &= \lambda(\partial_t u - X_H) \\ &= \lambda(\partial_t u) - \rho \cdot \alpha(h'(\rho) \cdot \mathcal{R}) \\ &= \lambda(\partial_t) - h'(\rho) \cdot \rho \\ \partial_t \rho = d(e^r)(\partial_t u) &= d(e^r)\big(J\partial_s u + X_H\big) \\ &= -\lambda(\partial_s u) + \underbrace{d(e^r)(X_H)}_{=0, \text{as orbits of } X_H \text{ stay in fixed } r\text{-levels}} \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad \Delta\rho &= \partial_s\big(\lambda(\partial_t u) - h'(\rho) \cdot \rho\big) - \partial_t \lambda(\partial_s u) \\ &= \partial_s \lambda(\partial_t u) - \partial_t \lambda(\partial_s u) - h'(\rho)\partial_s \rho - (\partial_s h')(\rho) \cdot \rho - h''(\rho) \cdot \rho \cdot \partial_s \rho \\ &= d\lambda(\partial_s u, \partial_t u) - \lambda\big(\underbrace{[\partial_s u, \partial_t u]}_{=0}\big) - dH(\partial_s u) - (\partial_s h')(\rho) \cdot \rho - h''(\rho) \cdot \rho \cdot \partial_s \rho \\ &= \omega(\partial_s u, \partial_t u - X_H) - (\partial_s h')(\rho) \cdot \rho - h''(\rho) \cdot \rho \cdot \partial_s \rho \\ &= |\partial_s u|^2 - (\partial_s h')(\rho) \cdot \rho - h''(\rho)(\rho) \cdot \rho \cdot \partial_s \rho \\ \Leftrightarrow \quad \Delta\rho + (h''(\rho) \cdot \rho) \cdot \partial_s \rho &\geq |\partial_s u|^2 - (\partial_s h')(\rho) \cdot \rho \geq 0. \end{aligned}$$

Now let $\Omega := u^{-1}\big(\Sigma \times (R + \varepsilon, \infty)\big)$. Due to the assumptions on the asymptotics of U, Ω is bounded. By Lemma 1 we find that ρ attains its maximum on $\partial\Omega$. Hence $u(s,t) \subset \widehat{V} \setminus \Sigma \times (R + \varepsilon, \infty)$ for all $\varepsilon > 0$. With $\varepsilon \to 0$, the proposition follows. □

2 First definition of symplectic homology

Now we give the definition of symplectic homology following Viterbo.

- The spectrum of (Σ, α) is $spec(\Sigma, \alpha) := \{l \mid \alpha \text{ has } \pm l\text{-periodic Reeb orbits}\} \cup \{0\}$

- Call a Hamiltonian admissible, writing $H \in Ad(\Sigma, \alpha)$, if it is cylindrical at infinity with
$$h(e^r) = \mathfrak{a} \cdot e^r + \mathfrak{b}, \text{ where } \mathfrak{a} \in \mathbb{R} \setminus spec(\Sigma, \alpha)$$
and all 1-periodic orbits are non-degenerate.

- Note that admissible Hamiltonians have only finitely many 1-periodic orbits. In view of the Maximum Principle we hence find that $FH_*(H)$ is well-defined. However, these groups depend strongly on H. Note that with the Maximum Principle, we get connecting homomorphisms $FH_*(H_+) \to FH_*(H_-)$ only if $H_- \geq H_+$ on $\Sigma \times [R, \infty)$ for some large R. However, for $H_1 \leq H_2 \leq H_3$ we still have commutative diagrams
$$\begin{array}{ccc} FH_*(H_1) & \longrightarrow & FH_*(H_2)\,. \\ & \searrow \quad \swarrow & \\ & FH_*(H_3) & \end{array}$$

- Define a partial order $\prec$ on $Ad(\Sigma, \alpha)$ by $H_1 \prec H_2$ iff $H_1 \leq H_2$ on $\Sigma \times [R, \infty)$ for some large R. Then define
$$SH_*(V) := \varinjlim_{H \in Ad(\Sigma,\alpha)} FH_*(H).$$

3 Direct and inverse limits

- A direct set $(M, \prec)$ is a set M with a partial order $\prec$ such that for each pair $\alpha, \beta \in M$ there exists $\gamma \in M$ with $\alpha, \beta \prec \gamma$ (Example $(Ad(\Sigma, \alpha), \prec)$).

- A subset $M' \subset M$ is cofinal if for every $\alpha \in M$ exists $\gamma \in M'$ such that $\alpha \prec \gamma$ (Ex. $H_n \in Ad(\Sigma, \alpha)$ with $\mathfrak{a}_n \to \infty$).

- A direct system of R-modules over $(M, \prec)$ consists of R-modules X^α $\forall \alpha \in M$ and R-linear maps $\iota^{\beta\alpha} : X^\alpha \to X^\beta \quad \forall \alpha \prec \beta$ such that $\iota^{\alpha\alpha} = id$, $\iota^{\gamma\alpha} = \iota^{\gamma\beta}\iota^{\beta\alpha} \quad \forall \alpha \prec \beta \prec \gamma$ (Ex. $(Ad(\Sigma, \alpha), \prec)$ with $X^H = FH(H)$).

- Let $Q \subset \bigoplus_{\alpha \in M} X^\alpha$ be a submodule generated by the elements $\iota^{\beta\alpha}x^\alpha - x^\alpha$ for any $\alpha \prec \beta$ and $x^\alpha \in X^\alpha$. Then
$$\varinjlim_{\alpha \in M} X^\alpha := \bigoplus_{\alpha \in M} X^\alpha \Big/ Q.$$
These are finite sums of elements in X^α, considered equal if they are eventually mapped to the same.

- An inverse system over $(M, \prec)$ consists of X_α together with R-linear maps $\pi_{\alpha\beta} : X_\beta \to X_\alpha \quad \forall \alpha \prec \beta$ such that $\pi_{\alpha\alpha} = id$, $\pi_{\alpha\gamma} = \pi_{\alpha\beta}\pi_{\beta\gamma} \quad \forall \alpha \prec \beta \prec \gamma$. Then define
$$\varprojlim_{\alpha \in M} X_\alpha := \left\{ (x_\alpha) \,\middle|\, \pi_{\alpha\beta}(x_\beta) = x_\alpha \forall \alpha \prec \beta \right\} \subset \prod_{\alpha \in M} X_\alpha.$$

- Facts: $\varinjlim$ is an exact functor, while $\varprojlim$ is only left exact, but exact when applied to finite dimensional vector spaces.

4 Action filtration

- SH has a qualitative and a quantitative aspect. This far, we considered only the quantitative feature.
- Assume that $[\omega]\pi_2(V) = 0$ or that $\omega = d\lambda$. Then we have a well-defined action on the loop space $\mathscr{L}(V)$ by

$$\mathcal{A}^H(x) = \int_{D^2} \bar{x}^*\omega - \int_0^1 H_t(x(t))dt,$$

  where $\bar{x} : D^2 \to V$ is such that $\bar{x}|_{S^1} = x$. Moreover, the Floer equation $(*)$ is the negative gradient equation of $\mathcal{A}^H$. It follows that the action increases from $-\infty$ to $+\infty$ along Floer cylinders $\Rightarrow \partial^F$ decreases action.
- Define for $b \notin spec(\Sigma, \alpha)$ the subcomplex $FC^{<b}(H) \subset FC(H)$ as generated by orbits x with $\mathcal{A}^H(x) < b$ and $FC^{(a,b)}(H) = FC^{<b}(H)\big/FC^{<a}(H)$. As ∂^F decreases action, it induces boundary operators on $FC^{<b}(H)$ and $FC^{(a,b)}(H) \Rightarrow FH^{<b}(H)$ and $FH^{(a,b)}(H)$.
- Let H be an everywhere monotone decreasing homotopy between $H_\pm$ and $x_\pm$ 1-periodic orbits of $H_\pm$. Then

$$\begin{aligned} \mathcal{A}^{H_+}(x_+) - \mathcal{A}^{H_-}(x_-) &= \int_{-\infty}^{\infty} \partial_s \mathcal{A}^{H_s}(u(s))ds \\ &= \int_{-\infty}^{\infty} ||\nabla \mathcal{A}^{H_s}||^2 ds - \int_{-\infty}^{\infty}\int_0^1 \big(\partial_s H\big)(u(s))dtds > 0. \end{aligned}$$

  So for globally decreasing homotopies the connecting homomorphism restricts to maps

$$FH^{<b}(H_+) \to FH^{<b}(H_-) \qquad \text{and} \qquad FH^{(a,b)}(H_+) \to FH^{(a,b)}(H_-).$$

- Now call H filtration admissible, $H \in Ad^0(\Sigma, \alpha)$, if $H \in Ad(\Sigma, \alpha)$ and $H|_V \leq 0$. Define a partial order $\leq$ on $Ad^0(\Sigma, \alpha)$ by $H_+ \leq H_-$ iff $H_+ \leq H_-$ globally as functions. Then define

$$SH^{<b}(V) = \varinjlim_{H \in Ad^0(\Sigma,\alpha)} FH^{<b}(H) \qquad \text{and} \qquad SH^{(a,b)}(V) = \varinjlim_{H \in Ad^0(\Sigma,\alpha)} FH^{(a,b)}(H).$$

- Inclusions $FC^{(a,b)}(H) \subset FC^{(a,b')}(H)$ and projections $FC^{(a',b)}(H) \subset FC^{(a,b)}(H)$ for $a' \leq a \leq b \leq b'$ induce maps in SH which give the groups $SH^{(a,b)}$ the structure of a bidirect system over $\mathbb{R} \times \mathbb{R}$. Fact:

$$\varinjlim_{b\to\infty} \varprojlim_{a\to-\infty} SH^{(a,b)}(V) \cong SH(V). \qquad \text{(with previous definition)}$$

  This holds as $SH^{(a,b)}(V) = SH^{(a',b)}$ for $a, a' < 0$ (consider cofinal sequence of Hamiltonians that are C^2-small inside V and cylindrical sharply increasing near Σ to final slope.) Hence suffices to show $\varinjlim SH^{(-\infty,b)}(V) = \varinjlim SH^{<b}(V) = SH(V)$. This is straight forward.

5 Variant

- By P. Seidel, one can take also one Hamiltonian H of the form $H(y,r) = h(e^r)$ with $\lim_{r\to\infty} h'(e^r) = \infty$. Then $FH(H) \cong SH(V)$.
- Has huge advantages when calculating SH (compare simplicial and singular homology).
- However, this does not recover action filtration and invariance follows only from the isomorphism with $SH(V)$. Moreover, we have cheated in the definition (see next chapter)

6 Morse-Bott

- In pratice, working only with non-degenerate orbits has a draw-back:
  - naturally, if H is autonomous, its orbits come in S^1-families, e.g. in areas where H is cylindrical, $H(y,r) = h(e^r)$, this is the case.
- Can we use this symmetry? Answer: Yes. Assumes that the 1-periodic orbits $\mathcal{N} := x(S^1)$ of X_H are isolated circles which are transversely non-degenerate, i.e. $\ker\left(D_p\phi_H^1 - Id\right) = T_p\mathcal{N}$. Now, there are two possibilities:
- Solution 1 (formal):
  perturb H with the help of a Morse-function f on $\mathcal{N} \cong S^1$ such that $\tilde{H} = H + \delta \cdot f$ has for δ small enough and each $\mathcal{N}$ two new constant 1-periodic orbits of degree $\mu_{CZ}(x) + \frac{1}{2}\left(1 \pm sign h''(r)\right)$ corresponding to the maximum and minimum of f.
- Solution 2 (flow lines with cascades):
  Chain complex is generated by critical points of f and differential counts flow lines with cascades, i.e. alternating sequences of whole Floer cylinders between different $\mathcal{N}$ and parts of Morse flow lines on $\mathcal{N}$ (see [3], appendix for precise definition)

7 Connection with topology

- Consider Hamiltonians as described previously - C^2-small Morse function on V and cylindrical sharply increasing near Σ (see image). So morally speaking $SH(V)$ is generated by critical points of H and two generators for each closed Reeb orbit (of arbitrary length).

- Explicitly $FH_*^{(-\infty,\varepsilon)}(H) \cong H_{*+n}(V, \partial V)$, as critical points of H have action close to zero and Floer trajectories are in one-to-one correspondence with Morse gradient trajectories of H. As the latter flow out of V along Σ, we get the relative homology.

- Moreover, $FH_*^{(-\infty,\varepsilon)}(H) \to SH_*^{(-\infty,\varepsilon)}(V) \to SH_*(V)$ by the natural map in direct limits and this gives a map $c : H_{*+n}(V, \partial V) \to SH_*(V)$.

8 The transfer map

- The map above is in fact natural and fits into a bigger frame: Let $(W, \partial W)$ be a 0-codimensional symplectic submanifold of $(V, \partial V)$ and assume that $V \setminus W$ is an exact symplectic manifold. Then there exists a natural homomorphims $i^!$ such that the following diagram commutes

$$\begin{array}{ccc} SH_*(V) & \xrightarrow{\quad i^! \quad} & SH_*(W) \\ \Big\uparrow c & & \Big\uparrow c \\ H_{*+n}(V, \partial V) & \xrightarrow{\quad i_* \quad} & H_{*+n}(W, \partial W) \end{array} \quad .$$

Construction idea: Consider Hamiltonians H of the shape

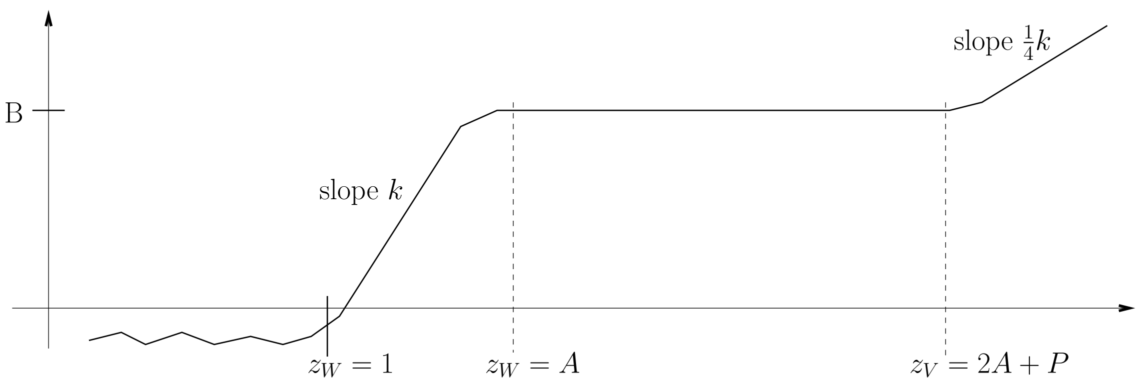

Fig. 1: Shape of H for transfer map

Then Gromov- Monotonicity assures that Floer trajectories inside W cannot escape and hence $SH^{\geq 0}(V) \cong SH(W)$. (see [1] and [2] for details)

9 Calculations

- Consider the unit ball $B_1(0) \subset \mathbb{C}^n$ and the standard symplectic structure $\omega := \frac{i}{2} dz \wedge d\bar{z}$. Take the standard Liouville form $\lambda = \frac{i}{4}(z d\bar{z} - \bar{z} dz)$ with Liouville vector field $Y(z) = \frac{1}{2} z \partial_z$, which generates the flow $\varphi^t(z) = e^{1/2t} \cdot z$. Note that $(\mathbb{C}^n, \omega)$ is the completion of $(B_1(0), \omega)$.

- Consider Hamiltonians $H_\alpha(z) = \alpha \cdot |z|^2 = \alpha \cdot z\bar{z}$, which are cylindrical, as for $z_0 \in S^{2n-1}$, we have $H(\varphi^t(z_0)) = e^t$. Their Hamiltonian vector fields are $X_H(z) = 2i\alpha z \cdot \partial_z$ with Hamiltonian flow $\varphi^t_{X_H}(z) = e^{2i\alpha t} \cdot z$. $\Rightarrow$ for $\alpha \notin \pi\mathbb{Z}$ is 0 the only 1-periodic orbit of X_H.

- The Conley-Zehnder index of the constant orbit γ_0 is

$$\mu_{CZ}(\gamma_0) = n \cdot \left(\left\lceil \frac{\alpha}{\pi} \right\rceil + \left\lfloor \frac{\alpha}{\pi} \right\rfloor \right) \overset{\alpha \to \infty}{\longrightarrow} \infty.$$

  Hence $FH_k(H_\alpha) = 0$ for α large enough $\Rightarrow SH_k(B_1(0)) = 0$.

- **Handle attachment**
  Let $V = W \cup_{\partial W} H^{2n}_k$ with $k < n$. Then

  Theorem (Cieliebak,[1]). $SH_*(V) \cong SH_*(W)$

  Proof: Idea: Consider the transfer map $SH_*(V) \to SH_*(W) = SH^{\geq 0}(V)$, which fits in the long exact sequence

$$SH^{<0}_*(V) \to SH_*(V) \to SH^{\geq 0}_*(V) \to SH^{<0}_{*-1}(V).$$

  Then show that $SH^{<0}_*(V) = 0$. This follows as one can construct Hamiltonians on V which have 1-periodic orbits below a certain action only in W or on the handle and on the handle a similar argument as for the ball shows that they do not count. □

  Corollary. *Let V be a subcritical Stein manifold, i.e. let V be obtained from the balll $B_1(0)$ by inductively adding a finite number of subcritical handles H^{2n}_k. Then $SH_*(V) = 0$.*

- **Viterbo's Theorem**
  Let M be any orientable smooth manifold. Consider the cotangent bundle T^*M. Points in T^*M are denoted (q,p) with $q \in M, p \in T^*_q M$. T^*M has a canonical Liouville form $\theta = pdq$, i.e. $\omega = d\theta = dp \wedge dq$ is symplectic. The choice of a Riemannian metric defines the unit disc bundle $D^*M := \{(q,p) \in T^*M \mid ||p|| \leq 1\}$. Let $\mathscr{L}M$ denote the free loop space of M.

  Theorem (Viterbo). $SH_*(D^*M, d\theta) = H_*(\mathscr{L}M)$

- The pair of pants product from Hamiltonian Floer homology carries over to symplectic homology. However, it does not restrict to the filtered version

10 Wrapped Floer homology or Lagrangian symplectic homology

- Let $(V, \lambda$ be a Liouville domain. Let $L \subset V$ be an exact Lagrangian which intersects ∂V transversely in a Legendrian submanifold $\partial L = L \cap \partial V$, i.e. $\lambda|_L$ is an exact 1-form which vanishes on ∂L.

- After applying a Hamiltonian isotopy, we may additionally assume that L is invariant under the flow of Y near ∂V, i.e. in a collar neighbourhood $\partial V \times (-\varepsilon, 0]$ L is identified with $\partial \times (-\varepsilon, 0]$.

- Take $H \in Ad(\partial V, \alpha)$. $FC(L)$ is generated by 1-periodic Hamiltonian chords (starting and ending on L), i.e. trajectories x of X_H with $x(0), x(1) \in L$. A chord x is non-degenerate if
  - 1 is not an eigenvalue of the linearized flow for constant chords
  - the image of TL under the linearized time 1 flow is transversal to TL, i.e. $D_{x(0)}\varphi^1_{X_H}\big(T_{x(0)}L\big) \pitchfork T_{x(1)}L$.

- ∂^F counts solutions $u : \mathbb{R} \times [0,1] \to V$ to $(*)$ such that $u(\mathbb{R} \times \{0,1\}) \subset L$. Otherwise, we construction of the homology is completely analogue to SH.

11 References

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